technology areas that are being developed to take over, providing a risk-hedging mechanism. However, the disadvantage is that the other 8 technology areas are temporarily abandoned, and with global events consistently changing, we may unknowingly have reduced our ability to handle a new global crisis.

Strategy C is to invest in the top 11 technology areas but scale them so that more important technologies get a proportionately higher percentage of the overall investment funding. The advantage is that all technology areas will be funded and developed, creating a portfolio effect for downstream strategic options of newer applications, and mitigating any downside risks of failures of any one technology. However, the disadvantage is that the critical need for global collaboration will be significantly delayed as focus is diffused over many technology areas.

Each of these simple example strategic paths has exit points and each also has an option of whether the technology should be tackled in-house or by some large integrator such as The Boeing Company or by smaller vendors with other expertise in these areas. These are nested options or options within options.

Of course the efforts are ongoing and would pose rather significant analytical and resource challenges. However, with the combinations of simulation, real options, systems dynamics, and optimization tools, the analysis methodology and results can become more valid and robust.

CASE STUDY: VALUING EMPLOYEE STOCK OPTIONS UNDER THE 2004 FAS 123R

This case study is based on Dr. Johnathan Mun's Valuing Employee Stock Options: Under 2004 FAS 123R *(Wiley Finance, 2004). This case study and book applies the same software FASB used to create the valuation examples in FAS 123R's section A87. It was this software application and the training seminars provided by the author for the Board of Directors at FASB, and one-on-one small group trainings for the project managers and research fellows at FASB, that convinced FASB of the pragmatic applications of employee stock options (ESO) valuation. The author consulted for and taught FASB about ESO valuation and is also the creator of the ESO Valuation Toolkit software used by FASB as well as many corporations and consultants.*

Executive Summary

In what the *Wall Street Journal* calls "among the most far-reaching steps that the Financial Accounting Standards Board (FASB) has made in its 30 year history,"[8] in December 2004 FASB released a final revised Statement of Financial Accounting Standard 123 (FAS 123R, or simply denoted as FAS 123) on Share-Based Payment amending the old FAS 123 and 95 issued in October 1995.[9] Basically, the proposal states that starting June 15, 2005, all new and portions of existing employee stock option (ESO) awards that have not yet vested will have to be expensed. In anticipation of the Standard, many companies such as GE and Coca-Cola had already voluntarily expensed their ESOs at the time of writing, while hundreds of other firms were scrambling to look into valuing their ESOs.

The goal of this case study is to provide the reader a better understanding of the valuation applications of FAS 123's preferred methodology—the binomial lattice—through a systematic and objective assessment of the methodology and comparing its results with the Black–Scholes model (BSM). This case study shows that, with care, FAS 123 valuation can be implemented accurately. The analysis performed uses a customized binomial lattice that takes into account real-life conditions such as vesting, employee suboptimal exercise behavior, forfeiture rates, and blackouts, as well as changing dividends, risk-free rates, and volatilities over the life of the ESO. This case study introduces the FAS 123 concept, followed by the different ESO valuation methodologies (closed-form BSM, binomial lattices, and Monte Carlo simulation) and their impacts on valuation. It is shown here that by using the right methodology that still conforms to the FAS 123 requirements, firms can potentially reduce their expenses by millions of dollars a year by avoiding the unnecessary overvaluation of the naïve BSM, using instead a modified and customized binomial lattice model that takes into account suboptimal exercise behavior, forfeiture rates, vesting, blackout dates, and changing inputs over time.

Introduction

The binomial lattice is the preferred method of calculating the fair-market valuation of ESOs in the FAS 123 requirements, but critics argue that companies do not necessarily have the resources in-house or the data availability to perform complex valuations that are both consistent with these new requirements and still be able to pass an audit. Based on a prior published study by the author that was presented to the FASB Board in 2003, it is concluded that the BSM, albeit theoretically correct and elegant, is insufficient and inappropriately applied when it comes to quantifying the fair-market value of ESOs.[10] This is because the BSM is applicable only to European options without dividends, where the holder of the option can exercise the option only on its maturity date and the underlying stock does not pay any dividends.[11] However, in reality, most ESOs are American-type[12] options with dividends, where the option holder can execute the option at any time up to and including the maturity date while the underlying stock pays dividends. In addition, under real-world conditions, ESOs have a time to *vesting* before the employee can execute the option, which may also be contingent on the firm and/or the individual employee attaining a specific performance level (e.g., profitability, growth rate, or stock price hitting a minimum barrier before the options become live), and subject to *forfeitures* when the employee leaves the firm or is terminated prematurely before reaching the vested period. In addition, certain options follow a *tranching* or graduated scale, where a certain percentage of the stock option grants become exercisable every year.[13] Also, employees exhibit erratic exercise behavior where the option will be executed only if it exceeds a particular multiple of the strike price; this is termed the *suboptimal exercise behavior multiple*. Next, the option value may be sensitive to the expected economic environment, as characterized by the term structure of interest rates (i.e., the U.S. Treasuries yield curve) where the risk-free rate changes during the life of the option. Finally, the firm may undergo some corporate restructuring (e.g., divestitures, or mergers and acquisitions that may require a stock swap that changes the volatility of the underlying stock). All these real-life scenarios make the BSM insufficient and inappropriate when used to place a fair-market value on the option grant.[14] In summary, firms can implement a variety of provisions that affect the fair value of the options. The closed-form models such as the BSM or the Generalized Black–Scholes (GBM)—the latter accounts for the inclusion of dividend yields—are inflexible and cannot be modified to accommodate these real-life conditions. Hence, the binomial lattice approach is preferred.

Under very specific conditions (European options without dividends) the binomial lattice and Monte Carlo simulation approaches yield identical values to the BSM, indicating that the two former approaches are robust and exact at the limit. However, when specific real-life business conditions are modeled (i.e., probability of forfeiture, probability the employee leaves or is terminated, time-vesting, suboptimal exercise behavior, and so forth), only the binomial lattice with its highly flexible nature will provide the true fair-market value of the ESO. The BSM takes into account only the following inputs: stock price, strike price, time to maturity, a single risk-free rate, and a single volatility. The GBM accounts for the same inputs as well as a single dividend rate. Hence, in accordance to the FAS 123 requirements, the BSM and GBM fail to account for real-life conditions. In contrast, the binomial lattice can be customized to include the stock price, strike price, time to maturity, a single risk-free rate and/or multiple risk-free rates changing over time, a single volatility and/or multiple volatilities changing over time, a single dividend rate and/or multiple dividend rates changing over time, plus all the other real-life factors including, but not limited to, vesting periods, suboptimal early exercise behavior, blackout periods, forfeiture rates, stock price and performance barriers, and other exotic contingencies. Note that the binomial lattice results revert to the GBM if these real-life conditions are negligible.

The two most important and convincing arguments for using binomial lattices are (1) that FASB requires it and states that the binomial lattice is the preferred method for ESO valuation and (2) that lattices can substantially reduce the cost of the ESO by more appropriately mirroring real-life conditions. Here is a sample of FAS 123's requirements discussing the use of binomial lattices.

> *B64. As discussed in paragraphs A10–A17, closed-form models are one acceptable technique for estimating the fair value of employee share options. However, a lattice model (or other valuation technique, such as a Monte Carlo simulation technique, that is not based on a closed-form equation) can accommodate the term structures of risk-free interest rates and expected volatility, as well as expected changes in dividends over an option's contractual term. A lattice model also can accommodate estimates of employees' option exercise patterns and post-vesting employment termination during the option's contractual term, and thereby can more fully reflect the effect of those factors than can an estimate developed using a closed-form model and a single weighted-average expected life of the options.*

> *A15. The Black–Scholes–Merton formula assumes that option exercises occur at the end of an option's contractual term, and that expected volatility, expected dividends, and risk-free interest rates are constant over the option's term. If used to estimate the fair value of instruments in the scope of this Statement, the Black–Scholes–Merton formula must be adjusted to take account of certain characteristics of employee share options and similar instruments that are not consistent with the model's assumptions (for example, the ability to exercise before the end of the option's contractual term). Because of the nature of the formula, those adjustments take the form of weighted average assumptions about those characteristics. In contrast, a lattice model can be designed to accommodate dynamic assumptions of expected volatility and dividends over the option's contractual term, and estimates of expected option exercise patterns during the option's contractual term, including the effect of blackout periods. Therefore, the design of a lattice model more fully reflects the substantive characteristics of a particular employee share option or similar instrument. Nevertheless, both a lattice model and the Black–Scholes–Merton formula, as well as other valuation techniques that meet the requirements in paragraph A8, can provide a fair value estimate that is consistent with the measurement objective and fair-value-based method of this Statement. However, if an entity uses a lattice*

model that has been modified to take into account an option's contractual term and employees' expected exercise and post-vesting employment termination behavior, the expected term is estimated based on the resulting output of the lattice. For example, an entity's experience might indicate that option holders tend to exercise their options when the share price reaches 200 percent of the exercise price. If so, that entity might use a lattice model that assumes exercise of the option at each node along each share price path in a lattice at which the early exercise expectation is met, provided that the option is vested and exercisable at that point. Moreover, such a model would assume exercise at the end of the contractual term on price paths along which the exercise expectation is not met but the options are in-the-money at the end of the contractual term. That method recognizes that employees' exercise behavior is correlated with the price of the underlying share. Employees' expected post-vesting employment termination behavior also would be factored in. Expected term, which is a required disclosure (paragraph A240), then could be estimated based on the output of the resulting lattice.

In fact, some parts of the FAS 123 Final Requirements cannot be modeled with a traditional Black–Scholes model. A lattice is required to model items such as suboptimal exercise behavior multiple, forfeiture rates, vesting, blackout periods, and so forth. This case study and the software used to compute the results use both a binomial (and trinomial) lattice as well as closed-form Black–Scholes models to compare the results. The specific FAS 123 paragraphs describing the use of lattices include:

A27. However, if an entity uses a lattice model that has been modified to take into account an option's contractual term and employees' expected exercise and post-vesting employment termination behavior, the expected term is estimated based on the resulting output of the lattice. For example, an entity's experience might indicate that option holders tend to exercise their options when the share price reaches 200 percent of the exercise price. If so, that entity might use a lattice model that assumes exercise of the option at each node along each share price path in a lattice at which the early exercise expectation is met, provided that the option is vested and exercisable at that point.

A28. Other factors that may affect expectations about employees' exercise and post-vesting employment termination behavior include the following:

a. The vesting period of the award. An option's expected term must at least include the vesting period.
b. Employees' historical exercise and post-vesting employment termination behavior for similar grants.

c. Expected volatility of the price of the underlying share.
d. Blackout periods and other coexisting arrangements such as agreements that allow for exercise to automatically occur during blackout periods if certain conditions are satisfied.
e. Employees' ages, lengths of service, and home jurisdictions (that is, domestic or foreign).

Therefore, based on the preceding justifications, and in accordance to the requirements and recommendations set forth by the revised FAS 123, which prefers the binomial lattice, it is hereby concluded that the customized binomial lattice is the best and preferred methodology to calculate the fair-market value of ESOs.

Application of the Preferred Method

In applying the customized binomial lattice methodology, several inputs have to be determined:

- Stock price at grant date.
- Strike price of the option grant.
- Time to maturity of the option.
- Risk-free rate over the life of the option.
- Dividend yield of the option's underlying stock over the life of the option.
- Volatility over the life of the option.
- Vesting period of the option grant.
- Suboptimal exercise behavior multiples over the life of the option.
- Forfeiture and employee turnover rates over the life of the option.
- Blackout dates postvesting when the options cannot be exercised.

The analysis assumes that the employee cannot exercise the option when it is still in the vesting period. Further, if the employee is terminated or decides to leave voluntarily during this vesting period, the option grant will be forfeited and presumed worthless. In contrast, after the options have been vested, employees tend to exhibit erratic exercise behavior where an option will be exercised only if it breaches the suboptimal exercise behavior multiple.[15] However, the options that have vested must be exercised within a short period if the employee leaves voluntarily or is terminated, regardless of the suboptimal behavior threshold—that is, if forfeiture occurs (measured by the historical option forfeiture rates as well as employee turnover rates). Finally, if the option expiration date has been reached, the option will be exercised if it is in-the-money, and expire worthless if it is at-the-money or out-of-the-money. The next section details the results obtained from such an analysis.

ESO Valuation Toolkit Software

It is theoretically impossible to solve a large binomial lattice ESO valuation without the use of software algorithms.[16] The analyses results in this case study were performed using the author's Employee Stock Options Valuation Toolkit 1.1 software (Figure 14.31), which is the same software used by FASB to convince itself that ESO valuation is pragmatic and manageable. In fact, FASB used this software to calculate the valuation example in the Final FAS 123 release in sections A87–A88 (illustrated later). Figure 14.32 shows a sample module for computing the Customized American Option using binomial lattices with vesting, forfeiture rate, suboptimal exercise behavior multiple, and changing risk-free rates and volatilities over time. The Real Options Super Lattice Solver software also can be used to create any customized ESO model using binomial lattices, FASB's favored method.

The software shows the applications of both closed-form models such as the BSM/GBM and binomial lattice methodologies. By using binomial lattice methodologies, more complex ESOs can be solved. For instance, the Customized Advanced Option (Figure 14.32) shows how multiple variables can be varied over time (risk-free, dividend, volatility, forfeiture rate, suboptimal exercise behavior multiple, and so forth). In addition, for added flexibility, the Super Lattice Solver module allows the expert user to create and solve his

FIGURE 14.31 ESO Valuation Toolkit 1.1 software.

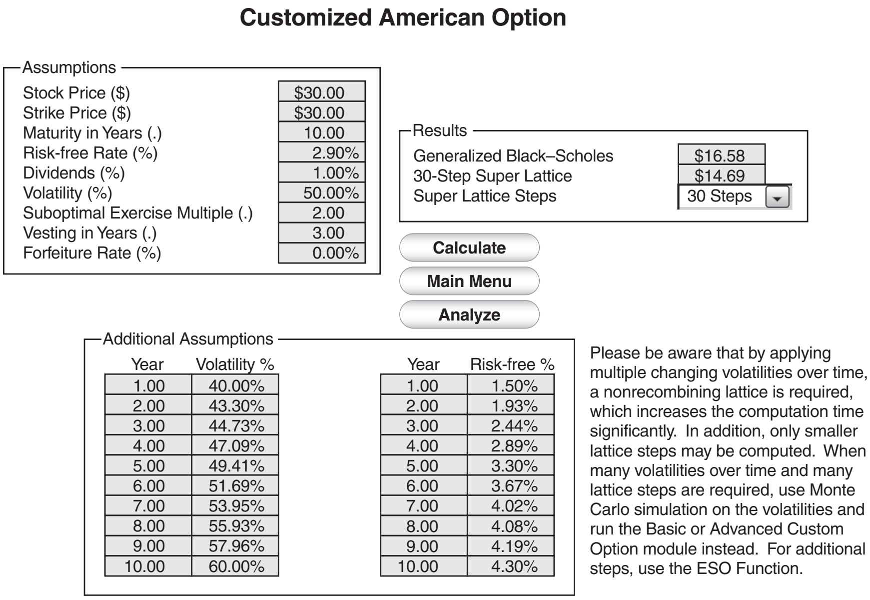

FIGURE 14.32 Customized advanced option model.

or her own customized ESO. This feature allows management to experiment with different flavors of ESO as well as to engineer one that would suit its needs, by balancing fair and equitable value to employees, with cost minimization to its shareholders.

Figure 14.32 shows the solution of the case example provided in section A87 of the Final 2004 FAS 123 standards. Specifically, A87–A88 state:

> *A87. The following table shows assumptions and information about the share options granted on January 1, 20X5.*
>
> *Share options granted 900,000*
> *Employees granted options 3,000*
> *Expected forfeitures per year 3.0%*
> *Share price at the grant date $30*
> *Exercise price $30*
> *Contractual term (CT) of options 10 years*
> *Risk-free interest rate over CT 1.5% to 4.3%*
> *Expected volatility over CT 40% to 60%*
> *Expected dividend yield over CT 1.0%*
> *Suboptimal exercise factor 2*

> *A88. This example assumes that each employee receives an equal grant of 300 options. Using as inputs the last 7 items from the table above, Entity T's lattice-based valuation model produces a fair value of $14.69 per option. A lattice model uses a suboptimal exercise factor to calculate the expected term (that is, the expected term is an output) rather than the expected term being a separate input. If an entity uses a Black–Scholes–Merton option-pricing formula, the expected term would be used as an input instead of a suboptimal exercise factor.*

Figure 14.32 shows the result as $14.69, the answer that FASB uses in its example. The forfeiture rate of 3 percent used by FASB's example is applied outside of the model to discount for the quantity reduced over time. The software allows the ability to input the forfeiture rates (pre- and post-vesting) inside or outside of the model. In this specific example, we set forfeiture rate to zero in Figure 14.32 and adjust the quantity outside, just as FASB does, in A91:

> *The number of share options expected to vest is estimated at the grant date to be 821,406* ($900{,}000 \times .97^3$).

In fact, using the ESO Valuation Toolkit software and Excel's goal seek function, we can find that the expected life of this option is 6.99 years. We can then justify the use of 6.99 years as the input into a modified GBM to obtain the same result at $14.69, something that cannot be done without the use of the binomial lattice approach.

Technical Justification of Methodology Employed

This section illustrates some of the technical justifications that make up the price differential between the GBM and the customized binomial lattice models. Figure 14.33 shows a tornado chart and how each input variable in a customized binomial lattice drives the value of the option.[17] Based on the chart, it is clear that volatility is not the single key variable that drives option value. In fact, when vesting, forfeiture, and suboptimal behavior elements are added to the model, their effects dominate that of volatility. The chart illustrated is based on a typical case and cannot be generalized across all cases.

In contrast, volatility is a significant variable in a simple BSM as can be seen in Figure 14.34. This is because there is less interaction among input variables due to the fewer input variables, and for most ESOs that are issued at-the-money, volatility plays an important part when there are no other dominant inputs.

In addition, the interactions among these new input variables are nonlinear. Figure 14.35 shows a spider chart[18] where it can be seen that vesting, forfeiture rates, and suboptimal exercise behavior multiples have nonlinear

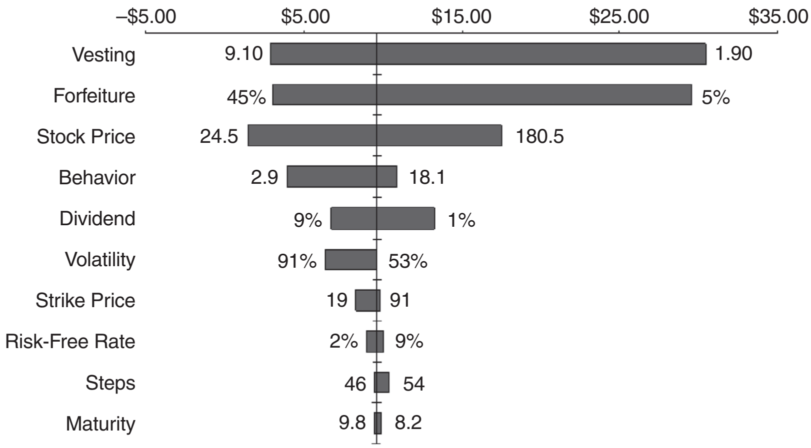

FIGURE 14.33 Tornado chart listing the critical input factors of a customized binomial model.

effects on option value. That is, the lines in the spider chart are not straight but curve at certain areas, indicating that there are nonlinear effects in the model. This means that we cannot generalize these three variables' effects on option value (for instance, we cannot generalize that if a 1 percent increase in forfeiture rate will decrease option value by 2.35 percent, it means that a 2 percent increase in forfeiture rate drives option value down 4.70 percent, and so forth). This is because the variables interact differently at different

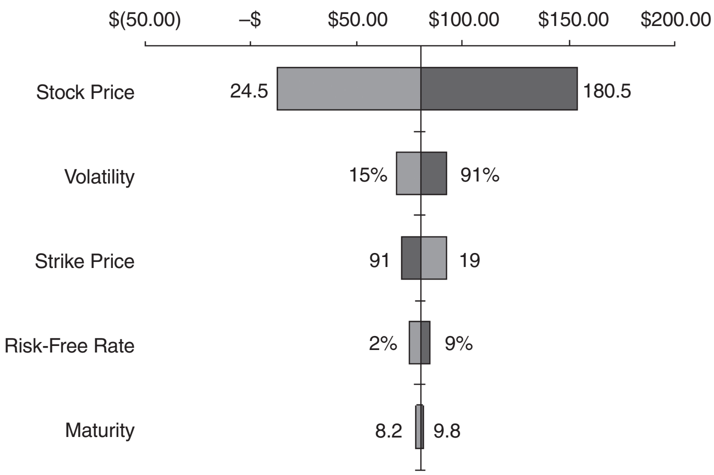

FIGURE 14.34 Tornado chart listing the critical input factors of the BSM.

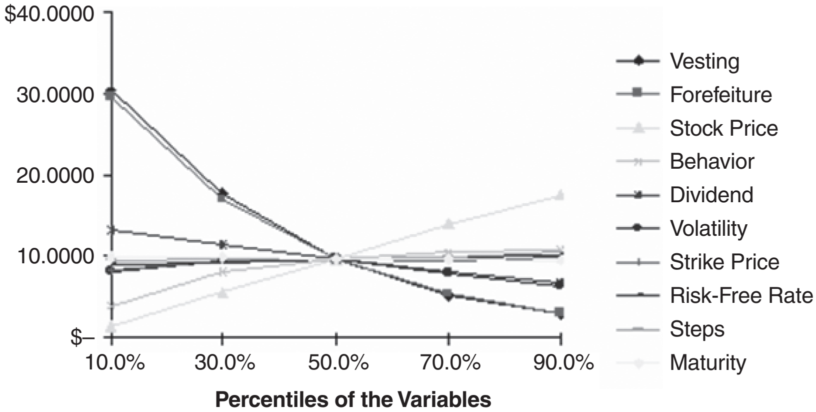

FIGURE 14.35 Spider chart showing the nonlinear effects of input factors in the binomial model.

input levels. The conclusion is that we really cannot say a priori what the direct effects are of changing one variable on the magnitude of the final option value. More detailed analysis will have to be performed in each case.

Although the tornado and spider charts illustrate the impact of each input variable on the final option value, the effects are static; that is, one variable is tweaked at a time to determine its ramifications on the option value. However, as shown, the effects are sometimes nonlinear, which means we need to change all variables simultaneously to account for their interactions. Figure 14.36 shows a Monte Carlo simulated dynamic sensitivity

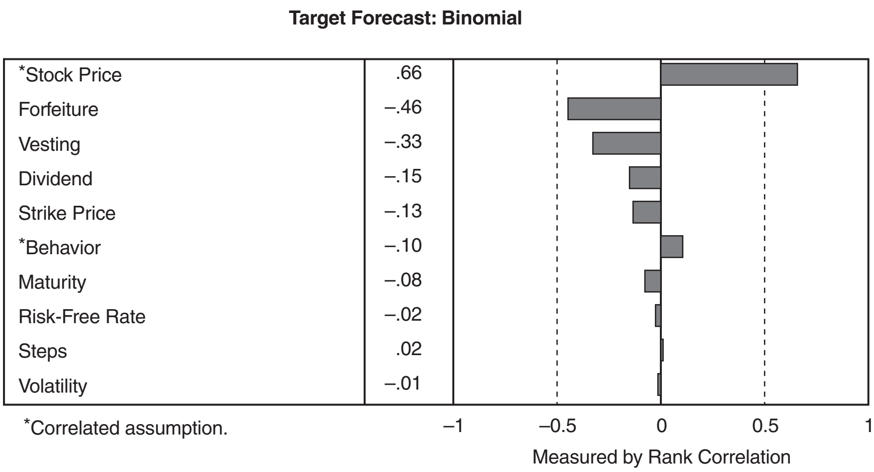

FIGURE 14.36 Dynamic sensitivity with simultaneously changing input factors in the binomial model.

chart where forfeiture, vesting, and suboptimal exercise behavior multiple are determined to be important variables, while volatility is again relegated to a less important role. The dynamic sensitivity chart perturbs all input variables simultaneously for thousands of trials, and captures the effects on the option value. This approach is valuable in capturing the net interaction effects among variables at different input levels.

From this preliminary sensitivity analysis, we conclude that incorporating forfeiture rates, vesting, and suboptimal exercise behavior multiple is vital to obtaining a fair-market valuation of ESOs due to their significant contributions to option value. In addition, we cannot generalize each input's effects on the final option value. Detailed analysis has to be performed to obtain the option's value every time.

Options with Vesting and Suboptimal Behavior

Further investigation into the elements of suboptimal behavior[19] and vesting yields the chart shown in Figure 14.37. Here we see that at lower suboptimal exercise behavior multiples (within the range of 1 to 6), the stock option value can be significantly lower than that predicted by the BSM. With a 10-year vesting stock option, the results are identical regardless of the suboptimal exercise behavior multiple—its flat line bears the same value as the BSM result. This is because for a 10-year vesting of a 10-year maturity option, the option reverts to a perfect European option, where it can be exercised only at expiration. The BSM provides the correct result in this case.

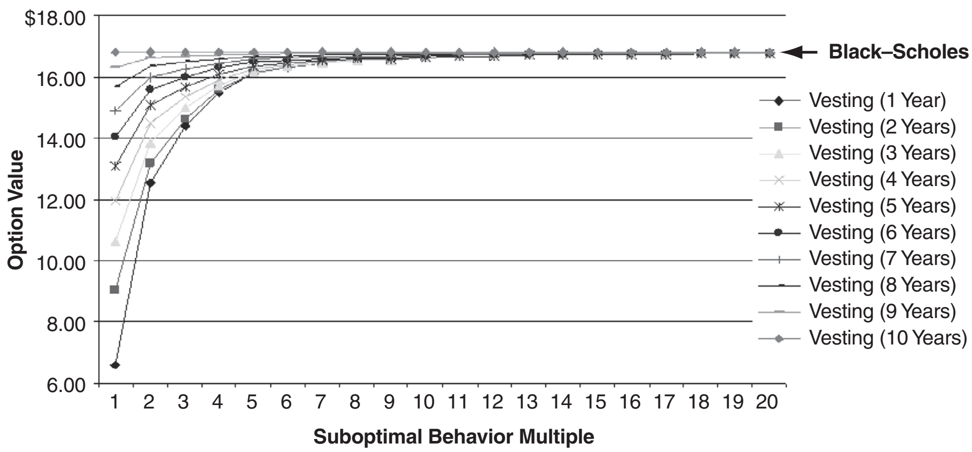

FIGURE 14.37 Impact of suboptimal exercise behavior and vesting on option value in the binomial model. (Assumptions used: stock and strike price of $25, 10-year maturity, 5% risk-free rate, 50% volatility, 0% dividends, suboptimal exercise behavior multiple range of 1–20, vesting period of 1–10 years, and tested with 100–5,000 binomial lattice steps.)

However, when suboptimal exercise behavior multiple is low, the option value decreases because employees holding the option will tend to exercise the option suboptimally—that is, the option will be exercised earlier and at a lower stock price than optimal. Hence, the option's upside value is not maximized. As an example, suppose an option's strike price is $10 while the underlying stock is highly volatile. If an employee exercises the option at $11 (this means a 1.10 suboptimal exercise multiple), he or she may not be capturing the entire upside potential of the option as the stock price can go up significantly higher than $11 depending on the underlying volatility. Compare this to another employee who exercises the option when the stock price is $20 (suboptimal exercise multiple of 2.0) versus one who does so at a much higher stock price. Thus, lower suboptimal exercise behavior means a lower fair-market value of the stock option. This suboptimal exercise behavior has a higher impact when stock prices at grant date are forecast to be high. Figure 14.38 shows that (at the lower end of the suboptimal multiples) a steeper slope occurs the higher the initial stock price at grant date.

Figure 14.39 shows that for higher volatility stocks, the suboptimal region is larger and the impact to option value is greater, but the effect is gradual. For instance, for the 100 percent volatility stock, the suboptimal region extends from a suboptimal exercise behavior multiple of 1.0 to approximately 9.0 versus from 1.0 to 2.0 for the 10 percent volatility stock. In addition, the vertical distance of the 100 percent volatility stock extends from

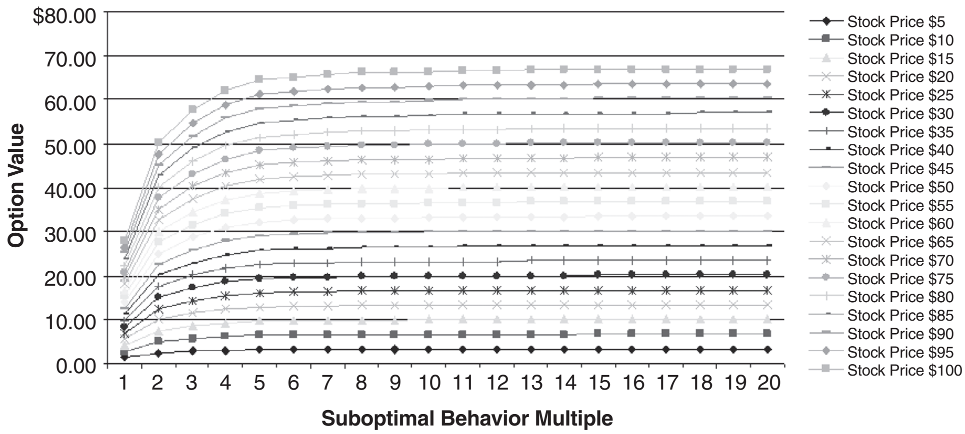

FIGURE 14.38 Impact of suboptimal exercise behavior and stock price on option value in the binomial model. (Assumptions used: stock and strike price range of $5 to $100, 10-year maturity, 5% risk-free rate, 50% volatility, 0% dividends, suboptimal exercise behavior multiple range of 1–20, 4-year vesting, and tested with 100–5,000 binomial lattice steps.)

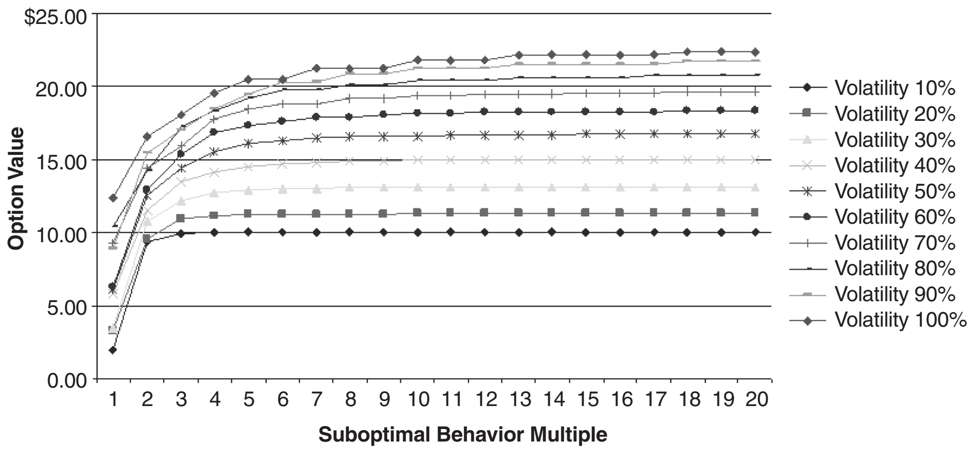

FIGURE 14.39 Impact of suboptimal exercise behavior and volatility on option value in the binomial model. (Assumptions used: stock and strike price of $25, 10-year maturity, 5% risk-free rate, 10–100% volatility range, 0% dividends, suboptimal exercise behavior multiple range of 1–20, 1-year vesting, and tested with 100–5,000 binomial lattice steps.)

$12 to $22 with a $10 range, as compared to $2 to $10 with an $8 range for the 10 percent volatility stock. Therefore, the higher the stock price at grant date and the higher the volatility, the greater the impact of suboptimal behavior will be on the option value. *In all cases,* the BSM results are the horizontal lines in the charts (Figures 14.38 and 14.39). That is, the BSM will always generate the maximum option value assuming optimal behavior, and overexpense the option significantly. A GBM or BSM cannot be modified to account for this suboptimal exercise behavior; only the binomial lattice can be used.

Options with Forfeiture Rates

Figure 14.40 illustrates the reduction in option value when the forfeiture rate increases. The rate of reduction changes depending on the vesting period. The longer the vesting period, the more significant the impact of forfeitures will be, illustrating once again the nonlinear interacting relationship between vesting and forfeitures (i.e., the lines in Figure 14.40 are curved and nonlinear). This is intuitive because the longer the vesting period, the lower the compounded probability that an employee will still be employed in the firm and the higher the chances of forfeiture, reducing the expected value of the option.

Again, we see that the BSM result is the highest possible value assuming a 10-year vesting in a 10-year maturity option with zero forfeiture (Figure

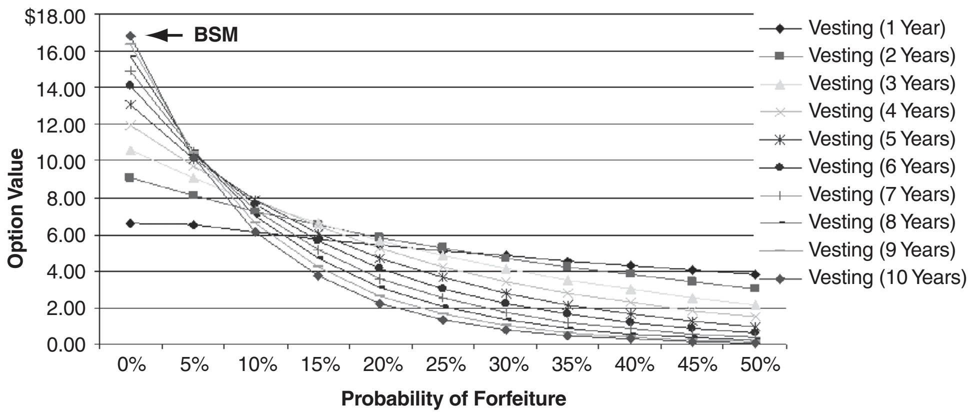

FIGURE 14.40 Impact of forfeiture rates and vesting on option value in the binomial model. (Assumptions used: stock and strike price of $25, 10-year maturity, 5% risk-free rate, 50% volatility, 0% dividends, suboptimal behavior 1.01, vesting period of 1–10 years, forfeiture range 0–50%, and tested with 100–5,000 binomial lattice steps.)

14.40). In addition, forfeiture rates can be negatively correlated to stock price—if the firm is doing well, its stock price usually increases, making the option more valuable and making the employees less likely to leave and the firm less likely to lay off its employees. Because the rate of forfeitures is uncertain (forfeiture rate fluctuations typically occur in the past due to business and economic environments, and will most certainly fluctuate again in the future) and is negatively correlated to the stock price, we can also apply a correlated Monte Carlo simulation on forfeiture rates in conjunction with the customized binomial lattices (shown later in this case study). The BSM will always generate the maximum option value assuming all options will fully vest and will overexpense the option significantly. The ESO Valuation software can account for forfeiture rates, while the accompanying Super Lattice Solver can account for different prevesting and postvesting forfeiture rates in the lattices.

Options Where Risk-Free Rate Changes Over Time

Another input assumption is the risk-free rate. Figure 14.41 illustrates the effects of changing risk-free rates over time on option valuation. When other exotic inputs are added, the changing risk-free lattice model has an overall lower valuation. In addition, due to the time value of money, discounting more heavily in the future will reduce the option's value. In other words, Figure 14.41 compares an upward sloping yield curve, a downward sloping

Basic Input Parameters		Year	Static Base Case	Increasing Risk-Free Rates	Decreasing Risk-Free Rates	Risk-Free Rate Smile	Risk-Free Rate Frown
Stock Price	$100.00	1	5.50%	1.00%	10.00%	8.00%	3.50%
Strike Price	$100.00	2	5.50	3.00	9.00	7.00	4.00
Maturity	10.00	3	5.50	3.00	8.00	5.00	5.00
Volatility	45.00	4	5.50	4.00	7.00	4.00	7.00
Dividend Rate	4.00	5	5.50	5.00	6.00	3.50	8.00
Lattice Steps	1000	6	5.50	6.00	5.00	3.50	8.00
Suboptimal Behavior	1.80	7	5.50	7.00	4.00	4.00	7.00
Vesting Period	4.00	8	5.50	8.00	3.00	5.00	5.00
Forfeiture Rate	10.00	9	5.50	9.00	2.00	7.00	4.00
		10	5.50	10.00	1.00	8.00	3.50
		Average	*5.50*	*5.50*	*5.50*	*5.50*	*5.50*
		BSM using 5.50% Average Rate	$37.45	$37.45	$37.45	$37.45	$37.45
		Forfeiture Modified BSM using 5.50% Average Rate	$33.71	$33.71	$33.71	$33.71	$33.71
		Changing Risk-free Binomial Lattice	$25.92	$24.31	$27.59	$26.04	$25.76

FIGURE 14.41 Effects of changing risk-free rates on option value. These results only illustrate a typical case and should not be generalized across all possible cases.

yield curve, risk-free rate smile, and risk-free rate frown. When the term structure of interest rates increases over time, the option value calculated using a customized changing risk-free rate binomial lattice is lower ($24.31) than that calculated using an average of the changing risk-free rates ($25.92) base case. The reverse is true for a downward-sloping yield curve. In addition, Figure 14.41 shows a risk-free yield curve frown (low rates followed by high rates followed by low rates) and a risk-free yield curve smile (high rates followed by low rates followed by high rates). The results indicate that using a single average rate will overestimate an upward-sloping yield curve, underestimate a downward-sloping yield curve, underestimate a yield curve smile, and overestimate a yield curve frown. Therefore, whenever appropriate, use all available information in terms of forward risk-free rates, one rate for each year.

Options Where Volatility Changes Over Time

Figure 14.42 illustrates the effects of changing volatilities on an ESO. If volatility changes over time, the BSM ($71.48) using the average volatility over time will *always* overestimate the true option value when there are other exotic inputs. In addition, compared to the $38.93 base case, slowly increasing volatilities over time from a low level has lower option values, while a decreasing volatility from high values and volatility smiles and frowns have higher values than using the average volatility estimate.

Basic Input Parameters		Year	Static Base Case	Increasing Volatilities	Decreasing Volatilities	Volatility Smile	Volatility Frown
Stock Price	$100.00	1	55.00%	10.00%	100.00$	80.00%	35.00%
Strike Price	$100.00	2	55.00	20.00	90.00	70.00	40.00
Maturity	10.00	3	55.00	30.00	80.00	50.00	50.00
Risk-free Rate	5.50	4	55.00	40.00	70.00	40.00	70.00
Dividend Rate	0.00	5	55.00	50.00	60.00	35.00	80.00
Lattice Steps	10	6	55.00	60.00	50.00	35.00	80.00
Suboptimal Behavior	1.80	7	55.00	70.00	40.00	40.00	70.00
Vesting Period	4.00	8	55.00	80.00	30.00	50.00	50.00
Forfeiture Rate	10.00	9	55.00	90.00	20.00	70.00	40.00
		10	55.00	100.00	10.00	80.00	35.00
		Average	*55.00*	*55.00*	*55.00*	*55.00*	*55.00*
		BSM using 5.50% Average Rate	$71.48	$71.48	$71.48	$71.48	$71.48
		Forfeiture Modified BSM using 5.50% Average Rate	$64.34	$64.34	$64.34	$64.34	$64.34
		Changing Risk-free Binomial Lattice	$38.93	$32.35	$45.96	$39.56	$39.71

FIGURE 14.42 Effects of changing volatilities on option value.

Options Where Dividend Yield Changes Over Time

Dividend yield is a simple input that can be obtained from corporate dividend policies or publicly available historical market data. It is the total dividend payments computed as a percentage of stock price that is paid out over the course of a year. The typical dividend yield is between 0 percent and 7 percent. In fact, about 45 percent of all publicly traded firms in the United States pay dividends. Of those that pay a dividend, 85 percent have a yield of 7 percent or below, and 95 percent have a yield of 10 percent or below.[20] Dividend yield is an interesting variable with very little interaction with other exotic input variables. It has a close to linear effect on option value, whereas the other exotic input variables do not. For instance, Figure 14.43 illustrates the effects of different maturities on the same option. The higher the maturity, the higher the option value, but the option value increases at a decreasing rate.

In contrast, Figure 14.44 illustrates the near-linear effects of dividends even when some of the exotic inputs have been changed. Whatever the change in variable is, the effects of dividends are always very close to linear. While Figure 14.44 illustrates many options with unique dividend rates, Figure 14.45 illustrates the effects of changing dividends over time on a single option. That is, the results shown in Figure 14.44 are based on comparing different options with different dividend rates, whereas the results shown in Figure 14.45 are based on a single option whose underlying stock's dividend yields are changing over the life of the option.

Maturity	Option Value	Change
1	$25.16	—
2	32.41	28.84%
3	35.35	9.08
4	36.80	4.08
5	37.87	2.91
6	38.41	1.44
7	38.58	0.43

FIGURE 14.43 Nonlinear effects of maturity. (Assumptions used: stock price and strike price are set at $100, 5% risk-free rate, 75% volatility, and 1,000 steps in the customized lattice, 1.8 behavior multiple, 1-year vesting, 10% forfeiture rate.)

	1.8 Behavior Multiple, 4-Year Vesting, 10% Forfeiture Rate		1.8 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate		3.0 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate	
Dividend Rate	Option Value	Change	Option Value	Change	Option Value	Change
0%	$42.15		$42.41		$49.07	
1	39.94	–5.24%	41.47	–2.20%	47.67	–2.86%
2	37.84	–5.27	40.55	–2.22	46.29	–2.89
3	35.83	–5.30	39.65	–2.24	44.94	–2.92
4	33.92	–5.33	38.75	–2.26	43.61	–2.95
5	32.10	–5.37	37.87	–2.28	42.31	–2.98

	$50 Stock Price, 1.8 Behavior Multiple, 1-Year Vesting, 10% Forfeiture Rate		1.8 Behavior Multiple, 1-Year Vesting, 5% Forfeiture Rate	
Dividend Rate	Option Value	Change	Option Value	Change
0%	$21.20		$45.46	
1	20.74	–2.20%	44.46	–2.20%
2	20.28	–2.22	43.47	–2.23
3	19.82	–2.24	42.49	–2.25
4	19.37	–2.26	41.53	–2.27
5	18.93	–2.28	40.58	–2.29

FIGURE 14.44 Near-linear effects of dividends.

Scenario	Option Value	Change	Notes
Static 3% Dividend	$39.65	0.00%	Dividends are kept steady at 3%
Increasing Gradually	$40.94	3.26%	1% to 5% with 1% increments (average of 3%)
Decreasing Gradually	$38.39	–3.17%	5% to 1% with –1% increments (average of 3%)
Increasing Jumps	$41.70	5.19%	0%, 0%, 5%, 5%, 5% (average of 3%)
Decreasing Jumps	$38.16	–3.74%	5%, 5%, 5%, 0%, 0% (average of 3%)

FIGURE 14.45 Effects of changing dividends over time. (Assumptions used: stock price and strike price are set at $100, 5-year maturity, 5% risk-free rate, 75% volatility, 1,000 steps in the customized lattice, 1.8 behavior multiple, 10% forfeiture rate, and 1-year vesting.)

Clearly, a changing-dividend option has some value to add in terms of the overall option valuation results. Therefore, if the firm's stock pays a dividend, then the analysis should also consider the possibility of dividend yields changing over the life of the option.

Options Where Blackout Periods Exist

Another item of interest is blackout periods, the dates that ESOs cannot be executed. These dates are usually several weeks before and several weeks after an earnings announcement (usually on a quarterly basis). In addition, only senior executives with fiduciary responsibilities have these blackout dates, and, hence, their proportion is relatively small compared to the rest of the firm. Figure 14.46 illustrates the calculations of a typical ESO with different blackout dates. In the case where there are only a few blackout days a month, there is little difference between options with blackout dates and those without blackout dates. In fact, if the suboptimal exercise behavior multiple is small (a 1.8 ratio is assumed in this case), blackout dates

Blackout Dates	*Option Value*
No Blackouts	$43.16
Every 2 years evenly spaced	43.16
First 5 years annual blackouts only	43.26
Last 5 years annual blackouts only	43.16
Every 3 months for 10 years	43.26

FIGURE 14.46 Effects of blackout periods on option value. (Assumptions used: stock and strike price of $100, 75% volatility, 5% risk-free rate, 10-year maturity, no dividends, 1-year vesting, 10% forfeiture rate, and 1,000 lattice steps.)

at strategic times will actually prevent the option holder from exercising suboptimally and sometimes even increase the value of the option ever so slightly.

The analysis shown as Figure 14.46 assumes only a small percentage of blackout dates in a year (e.g., during several days in a year, the ESO cannot be executed). This may be the case for certain so-called brick-and-mortar companies, and, as such, blackout dates can be ignored. However, in other firms such as those in the biotechnology and high-tech industries, blackout periods play a more significant role. For instance, in a biotech firm, blackout periods may extend 4–6 weeks every quarter, straddling the release of its quarterly earnings. In addition, blackout periods prior to the release of a new product may exist. Therefore, the proportion of blackout dates with respect to the life of the option may reach upward of 35–65 percent per year. In such cases, blackout periods will significantly affect the value of the option. For instance, Figure 14.47 illustrates the differences between a customized binomial lattice with and without blackout periods. By adding in the real-life elements of blackout periods, the ESO value is further reduced by anywhere between 10 percent and 35 percent depending on the rate of forfeiture and volatility. As expected, the reduction in value is nonlinear, as the effects of blackout periods will vary depending on the other input variables involved in the analysis.

% Difference between no blackout periods versus significant blackouts	Volatility (25%)	Volatility (30%)	Volatility (35%)	Volatility (40%)	Volatility (45%)	Volatility (50%)
Forfeiture Rate (5%)	–17.33%	–13.18%	–10.26%	–9.21%	–7.11%	–5.95%
Forfeiture Rate (6%)	–19.85%	–15.17%	–11.80%	–10.53%	–8.20%	–6.84%
Forfeiture Rate (7%)	–22.20%	–17.06%	–13.29%	–11.80%	–9.25%	–7.70%
Forfeiture Rate (8%)	–24.40%	–18.84%	–14.71%	–13.03%	–10.27%	–8.55%
Forfeiture Rate (9%)	–26.44%	–20.54%	–16.07%	–14.21%	–11.26%	–9.37%
Forfeiture Rate (10%)	–28.34%	–22.15%	–17.38%	–15.35%	–12.22%	–10.17%
Forfeiture Rate (11%)	–30.12%	–23.67%	–18.64%	–16.45%	–13.15%	–10.94%
Forfeiture Rate (12%)	–31.78%	–25.11%	–19.84%	–17.51%	–14.05%	–11.70%
Forfeiture Rate (13%)	–33.32%	–26.48%	–21.00%	–18.53%	–14.93%	–12.44%
Forfeiture Rate (14%)	–34.77%	–27.78%	–22.11%	–19.51%	–15.78%	–13.15%
Forfeiture Rate (14%)	–34.77%	–27.78%	–22.11%	–19.51%	–15.78%	–13.15%

FIGURE 14.47 Effects of significant blackouts (different forfeiture rates and volatilities). (Assumptions used: stock and strike price range of $30 to $100, 45% volatility, 5% risk-free rate, 10-year maturity, dividend range 0–10%, vesting of 1–4 years, 5–14% forfeiture rate, suboptimal exercise behavior multiple range of 1.8–3.0, and 1,000 lattice steps.)

% Difference between no blackout periods versus significant blackouts	Vesting (1)	Vesting (2)	Vesting (3)	Vesting (4)
Dividends (0%)	–8.62%	–6.93%	–5.59%	–4.55%
Dividends (1%)	–9.04%	–7.29%	–5.91%	–4.84%
Dividends (2%)	–9.46%	–7.66%	–6.24%	–5.13%
Dividends (3%)	–9.90%	–8.03%	–6.56%	–5.43%
Dividends (4%)	–10.34%	–.841%	–6.90%	–5.73%
Dividends (5%)	–10.80%	–8.79%	–7.24%	–6.04%
Dividends (6%)	–11.26%	–9.18%	–7.58%	–6.35%
Dividends (7%)	–11.74%	–9.58%	–7.93%	–6.67%
Dividends (8%)	–12.22%	–9.99%	–8.29%	–6.99%
Dividends (9%)	–12.71%	–10.40%	–8.65%	–7.31%
Dividends (10%)	–13.22%	–10.81%	–9.01%	–7.64%

FIGURE 14.48 Effects of significant blackouts (different dividend yields and vesting periods).

Figure 14.48 shows the effects of blackouts under different dividend yields and vesting periods, while Figure 14.49 illustrates the results stemming from different dividend yields and suboptimal exercise behavior multiples. Clearly, it is almost impossible to predict the exact impact unless a detailed analysis is performed, but the range can be generalized to be typically between 10 percent and 20 percent. Blackout periods can only be modeled in a binomial lattice and not in the BSM/GBM.

Nonmarketability Issues

The 2004 FAS 123 revision does not explicitly discuss the issue of nonmarketability; that is, ESOs are neither directly transferable to someone else nor freely tradable in the open market. Under such circumstances, it can be argued based on sound financial and economic theory that a nontradable and nonmarketable discount can be appropriately applied to the ESO. However, this is not a simple task.

A simple and direct application of a discount should not be based on an arbitrarily chosen percentage *haircut* on the resulting binomial lattice result. Instead, a more rigorous analysis can be performed using a *put option*. A call option is the contractual right, but not the obligation, to *purchase* the underlying stock at some predetermined contractual strike price within a specified time, while a put option is a contractual right, but not the obligation, to *sell* the underlying stock at some predetermined contractual price within a specified time. Therefore, if the holder of the ESO cannot sell or transfer the

% Difference between no blackout periods versus significant blackouts	Dividends (0%)	Dividends (1%)	Dividends (2%)	Dividends (3%)	Dividends (4%)	Dividends (5%)	Dividends (6%)	Dividends (7%)	Dividends (8%)	Dividends (9%)	Dividends (10%)
Suboptimal Behavior Multiple (1.8)	–1.01%	–1.29%	–1.58%	–1.87%	–2.16%	–2.45%	–2.75%	–3.06%	–3.36%	–3.67%	–3.98%
Suboptimal Behavior Multiple (1.9)	–1.01%	–1.29%	–1.58%	–1.87%	–2.16%	–2.45%	–2.75%	–3.06%	–3.36%	–3.67%	–3.98%
Suboptimal Behavior Multiple (2.0)	–1.87%	–2.29%	–2.72%	–3.15%	–3.59%	–4.04%	–4.50%	–4.96%	–5.42%	–5.90%	–6.38%
Suboptimal Behavior Multiple (2.1)	–1.87%	–2.29%	–2.72%	–3.15%	–3.59%	–4.04%	–4.50%	–4.96%	–5.42%	–5.90%	–6.38%
Suboptimal Behavior Multiple (2.2)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.34%
Suboptimal Behavior Multiple (2.3)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.34%
Suboptimal Behavior Multiple (2.4)	–4.71%	–5.05%	–5.39%	–5.74%	–6.10%	–6.46%	–6.82%	–7.19%	–7.57%	–7.95%	–8.23%
Suboptimal Behavior Multiple 92.5)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.7)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	–10.86%	–11.41%
Suboptimal Behavior Multiple (2.8)	–6.34%	–6.80%	–7.28%	–7.77%	–8.26%	–8.76%	–9.27%	–9.79%	–10.32%	10.86%	–11.41%
Suboptimal Behavior Multiple (2.9)	–8.62%	–9.04%	–9.46%	–9.9–%	–10.34%	–10.80%	–11.26%	–11.74%	–12.22%	–12.71%	–13.22%
Suboptimal Behavior Multiple (3.0)	–8.62%	–9.04%	–9.46%	–9.90%	–10.34%	–10.80%	–11.26%	–11.74%	–12.22%	–12.71%	–13.22%

FIGURE 14.49 Effects of significant blackouts (different dividend yields and exercise behaviors).

rights of the option to someone else, then the holder of the option has given up his or her rights to a put option (i.e., the employee has written or sold the firm a put option). Calculating the put option and discounting this value from the call option provides a theoretically correct and justifiable nonmarketability and nontransferability discount to the existing option.

However, care should be taken in analyzing this haircut or discounting feature. The same inputs that go into the customized binomial lattice to calculate a call option should also be used to calculate a customized binomial lattice for a put option. That is, the put option must also be under the same risks (volatility that can change over time), economic environment (risk-free rate structure that can change over time), corporate financial policy (a static or changing dividend yield over the life of the option), contractual obligations (vesting, maturity, strike price, and blackout dates), investor irrationality (suboptimal exercise behavior), firm performance (stock price at grant date), and so forth.

Although nonmarketability discounts or haircuts are not explicitly discussed in FAS 123, the valuation analysis is performed here for the sake of completeness. It is up to each firm's management to decide if haircuts should and can be applied. Figure 14.50 shows the customized binomial lattice valuation results of a typical ESO. Figure 14.51 shows the results from a nonmarketability analysis performed using a down-and-in upper barrier modified put option with the same exotic inputs (vesting, blackouts, forfeitures, suboptimal behavior, and so forth) calculated using the customized binomial lattice model.[21] The discounts range from 22 percent to 53 percent. These calculated discounts look somewhat significant but are actually in

Customized Binomial Lattice (Option Valuation)	Behavior (1.20)	Behavior (1.40)	Behavior (1.60)	Behavior (1.80)	Behavior (2.00)	Behavior (2.20)	Behavior (2.40)	Behavior (2.60)	Behavior (2.80)	Behavior (3.00)
Forfeiture (0.00%)	$24.57	$30.53	$36.16	$39.90	$43.15	$45.87	$48.09	$49.33	$50.40	$51.31
Forfeiture (4.00%)	$22.69	$27.65	$32.19	$35.15	$37.67	$39.74	$41.42	$42.34	$43.13	$43.80
Forfeiture (10.00%)	$21.04	$25.22	$28.93	$31.29	$33.27	$34.88	$36.16	$36.86	$37.45	$37.94
Forfeiture (15.00%)	$19.58	$23.13	$26.20	$28.11	$29.69	$30.94	$31.93	$32.46	$32.91	$33.29
Forfeiture (20.00%)	$18.28	$21.32	$23.88	$25.44	$26.71	$27.70	$28.48	$28.89	$29.23	$29.52
Forfeiture (25.00%)	$17.10	$19.73	$21.89	$23.17	$24.20	$25.00	$25.61	$25.93	$26.19	$26.41
Forfeiture (30.00%)	$16.02	$18.31	$20.14	$21.21	$22.06	$22.70	$23.19	$23.44	$23.65	$23.82
Forfeiture (35.00%)	$15.04	$17.04	$18.61	$19.51	$20.20	$20.73	$21.12	$21.32	$21.49	$21.62
Forfeiture (40.00%)	$14.13	$15.89	$17.24	$18.00	$18.58	$19.01	$19.33	$19.49	$19.63	$19.73

FIGURE 14.50 Customized binomial lattice valuation results. (Assumptions used: stock and strike price of $100, 10-year maturity, 1-year vesting, 35% volatility, 0% dividends, 5% risk-free rate, suboptimal exercise behavior multiple range of 1.2–3.0, forfeiture range of 0–40%, and 1,000 step customized lattice.)

Haircut (Customized Binomial Lattice Modified Put)	Behavior (1.20)	Behavior (1.40)	Behavior (1.60)	Behavior (1.80)	Behavior (2.00)	Behavior (2.20)	Behavior (2.40)	Behavior (2.60)	Behavior (2.80)	Behavior (3.00)
Forfeiture (0.00%)	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33	$11.33
Forfeiture (5.00%)	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76	$10.76
Forfeiture (10.00%)	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23	$10.23
Forfeiture (15.00%)	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72	$9.72
Forfeiture (20.00%)	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23	$9.23
Forfeiture (25.00%)	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77	$8.77
Forfeiture (30.00%)	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34	$8.34
Forfeiture (35.00%)	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92	$7.92
Forfeiture (40.00%)	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52	$7.52

Nonmarketability and Nontransferability Discount (%)	Behavior (1.20)	Behavior (1.40)	Behavior (1.60)	Behavior (1.80)	Behavior (2.00)	Behavior (2.20)	Behavior (2.40)	Behavior (2.60)	Behavior (2.80)	Behavior (3.00)
Forfeiture (0.00%)	46.09%	37.09%	31.32%	28.39%	26.25%	24.69%	23.55%	22.96%	22.47%	22.07%
Forfeiture (5.00%)	47.43%	38.92%	33.43%	30.62%	28.57%	27.08%	25.98%	25.42%	24.95%	24.57%
Forfeiture (10.00%)	48.60%	40.55%	35.35%	32.68%	30.73%	29.32%	28.28%	27.75%	27.31%	26.95%
Forfeiture (15.00%)	49.62%	42.01%	37.08%	34.57%	32.73%	31.40%	30.43%	29.93%	29.53%	29.19%
Forfeiture (20.00%)	50.52%	43.31%	38.66%	36.29%	34.57%	33.33%	32.42%	31.96%	31.59%	31.28%
Forfeiture (25.00%)	51.32%	44.48%	40.09%	37.86%	36.25%	35.10%	34.26%	33.84%	33.49%	33.22%
Forfeiture (30.00%)	52.03%	45.53%	41.38%	39.29%	37.79%	36.72%	35.95%	35.56%	35.25%	35.00%
Forfeiture (35.00%)	52.67%	46.48%	42.56%	40.60%	39.20%	38.21%	37.50%	37.15%	36.86%	36.63%
Forfeiture (40.00%)	53.24%	47.34%	43.64%	41.80%	40.49%	39.57%	38.92%	38.60%	38.34%	38.14%

FIGURE 14.51 Nonmarketability and nontransferability discount.

line with market expectations.[22] As these discounts are not explicitly sanctioned by FASB, the author cautions their use in determining the fair-market value of the ESOs.

Expected Life Analysis

As seen previously, the 2004 Final FAS 123 Sections A15 and B64 expressly prohibit the use of a modified BSM with a single expected life. This means that instead of using an expected life as the *input* into the BSM to obtain the similar results as in a customized binomial lattice, the analysis should be done the other way around. That is, using vesting requirements, suboptimal exercise behavior multiples, forfeiture or employee turnover rates, and the other standard option inputs, calculate the valuation results using the customized binomial lattice. This result can then be compared with a modified BSM and the expected life can then be *imputed*. Excel's goal-seek function can be used to obtain the imputed expected life of the option by setting the BSM result equal to the customized binomial lattice. The resulting expected life can then be compared with historical data as a secondary verification of the results, that is, if the expected life falls within reasonable bounds based on historical performance. This is the correct approach because measuring the expected life of an option is very difficult and inaccurate.

Figure 14.52 illustrates the use of Excel's goal-seek function on the ESO Valuation Toolkit software to impute the expected life into the BSM model by setting the BSM results equal to the customized binomial lattice results.

Figure 14.53 illustrates another case where the expected life can be imputed, but this time the forfeiture rates are not set at zero. In this case, the BSM results will need to be modified. For example, the customized binomial lattice result of $5.41 is obtained with a 15 percent forfeiture rate. This means that the BSM result needs to be BSM(1–15%) = $5.41 using the modified expected life method. The expected life that yields the BSM value of $6.36 ($5.41/85% is $6.36, and $6.36(1–15%) is $5.41) is 2.22 years.

Dilution

In most cases, the effects of dilution can be safely ignored as the proportion of ESO grants is relatively small compared to the total equity issued by the company. In investment finance theory, the market has already anticipated

Customized Binomial Lattice Results to Impute the Expected Life for BSM

Applying Different Suboptimal Behavior Multiples

Stock Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Strike Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Maturity	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Risk-Free Rate	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%
Dividend	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Volatility	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Vesting	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Suboptimal Behavior	**1.10**	**1.50**	**2.00**	**2.50**	**3.00**	**3.50**	**4.00**
Forfeiture Rate	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Lattice Steps	1000	1000	1000	1000	1000	1000	1000
Binomial	$8.94	$10.28	$11.03	$11.62	$11.89	$12.18	$12.29
BSM	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87
Expected Life	***4.42***	***5.94***	***6.95***	***7.83***	***8.26***	***8.74***	***8.93***
Modified BSM	$8.94	$10.28	$11.03	$11.62	$11.89	$12.18	$12.29

FIGURE 14.52 Imputing the expected life for the BSM using the binomial lattice results.

Customized Binomial Lattice Results to Impute the Expected Life for BSM
Applying Different Forfeiture Rates

Stock Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Strike Price	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00	$20.00
Maturity	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Risk-Free Rate	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%	3.50%
Dividend	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Volatility	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Vesting	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Suboptimal Behavior	1.50	1.50	1.50	1.50	1.50	1.50	1.50
Forfeiture Rate	**0.00%**	**.250%**	**5.00%**	**7.50%**	**10.00%**	**12.50%**	**15.00%**
Lattice Steps	1000	1000	1000	1000	1000	1000	1000
Binomial	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41
BSM	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87	$12.87
Expected Life	***5.94***	***4.71***	***3.77***	***3.03***	***2.45***	***1.99***	***1.61***
Modified BSM*	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41
Expected Life	***5.94***	***4.97***	***4.19***	***3.55***	***3.02***	***2.59***	***2.22***
Modified BSM**	$10.28	$9.23	$8.29	$7.44	$6.69	$6.02	$5.41

*Note: Uses the binomial lattice result to impute the expected life for a modified BSM.
**Note: Uses the binomial lattice but also accounts for the Forfeiture rate to modify the BSM.

FIGURE 14.53 Imputing expected life for the BSM using lattice results under nonzero forfeiture rates.

the exercise of these ESOs and the effects have already been accounted for in the stock price. Once a new grant is announced, the stock price will immediately and fully incorporate this news and account for any dilution that may occur. This means that as long as the valuation is performed after the announcement is made, then the effects of dilution are nonexistent. The 2004 FAS 123 revisions do not explicitly provide guidance in this area. Given that FASB provides little guidance on dilution (Section A39), and because forecasting stock prices (as part of estimating the effects of dilution) is fairly difficult and inaccurate at best, plus the fact that the dilution effects are minimal (small in proportion compared to all the equity issued by the firm), the effects of dilution are assumed to be minimal and can be safely ignored.

Applying Monte Carlo Simulation for Statistical Confidence and Precision Control

Next, Monte Carlo simulation can be applied to obtain a range of calculated stock option fair values. That is, any of the inputs into the stock options valuation model can be chosen for Monte Carlo simulation if they are uncertain and stochastic. Distributional assumptions are assigned to these variables, and the resulting option values using the BSM, GBM, path simulation, or binomial lattices are selected as forecast cells. These modeled uncertainties include the probability of forfeiture and the employees' suboptimal exercise behavior.

The results of the simulation are essentially a distribution of the stock option values. Keep in mind that the simulation application here is used to vary the inputs to an options valuation model to obtain a range of results, not to model and calculate the options themselves. However, simulation can be applied both to simulate the inputs to obtain the range of options results and to solve the options model through path-dependent simulation. For instance, the simulated input assumptions are those inputs that are highly uncertain and can vary in the future, such as stock price at grant date, volatility, forfeiture rates, and suboptimal exercise behavior multiples. Clearly, variables that are objectively obtained, such as risk-free rates (U.S. Treasury yields for the next 1 month to 20 years are published), dividend yield (determined from corporate strategy), vesting period, strike price, and blackout periods (determined contractually in the option grant) should not be simulated. In addition, the simulated input assumptions can be correlated. For instance, forfeiture rates can be negatively correlated to stock price—if the firm is doing well, its stock price usually increases, making the option more valuable, thus making the employees less likely to leave and the firm less likely to lay off its employees. Finally, the output forecasts are the option valuation results. In fact, Monte Carlo simulation is allowed and recommended in FAS 123 (Sections B64, B65, and footnotes 48, 52, 74, and 97).

Figure 14.54 shows the results obtained using the customized binomial lattices based on single-point inputs of all the variables. The model takes exotic inputs such as vesting, forfeiture rates, suboptimal exercise behavior multiples, blackout periods, and changing inputs (dividends, risk-free rates, and volatilities) over time. The resulting option value is $31.42. This analysis can then be extended to include simulation. Figure 14.55 illustrates the use of simulation coupled with customized binomial lattices (Risk Simulator software was used to simulate the input variables).

Rather than randomly deciding on the correct number of trials to run in the simulation, statistical significance and precision control are set up to run the required number of trials automatically. A 99.9 percent statistical confidence on a $0.01 error precision control was selected and 145,510 simulation trials were run.[23] This highly stringent set of parameters means that an adequate number of trials will be run to ensure that the results will fall within a $0.01 error variability 99.9 percent of the time. For instance, the simulated average result was $31.32 (Figure 14.55). This means that 999

Risk-Free Rate		*Volatility*		*Dividend Yield*		*Suboptimal Behavior*	
Year	Rate	Year	Rate	Year	Rate	Year	
1	3.50%	1	35.00%	1	1.00%	1	1.80
2	3.75	2	35.00	2	1.00	2	1.80
3	4.00	3	35.00	3	1.00	3	1.80
4	4.15	4	45.00	4	1.50	4	1.80
5	4.20	5	45.00	5	1.50	5	1.80

		Forfeiture Rate		*Blackout Dates*	
		Year	Rate	Month	Step
Stock Price	$100	1	5.00%	12	12
Strike Price	$100	2	5.00	24	24
Time to Maturity	5	3	5.00	36	36
Vesting Period	1	4	5.00	48	48
Lattice Steps	60	5	5.00	60	60
Option value	**$31.42**				

FIGURE 14.54 Single-point result using a customized binomial lattice.

Statistic	Value	Precision
Trials	145,510	
Mean	$31.32	$0.01
Median	$31.43	$0.02
Mode	—	
Standard Deviation	$1.57	$0.01
Variance	$2.46	
Skewness	−0.21	
Kurtosis	2.43	
Coeff. Of Variability	0.05	
Range Minimum	$26.59	
Range Maximum	$35.62	
Range Width	$9.03	
Mean Std. Error	$0.00	

*Tested for $0.01 precision at 99.90% confidence.

FIGURE 14.55 Options valuation result at $0.01 precision with 99.9 percent confidence.

out of 1,000 times, the true option value will be accurate to within $0.01 of $31.32. These measures are statistically valid and objective.[24]

Number of Steps

The higher the number of lattice steps, the higher the precision of the results. Figure 14.56 illustrates the convergence of results obtained using a BSM closed-form model on a European call option without dividends, and comparing its results to the basic binomial lattice. Convergence is generally achieved at 1,000 steps. As such, the analysis results will use 1,000 steps

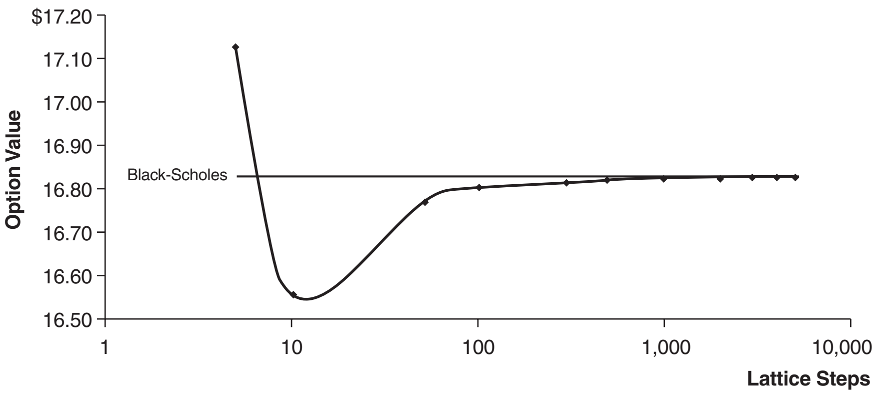

FIGURE 14.56 Convergence of the binomial lattice to closed-form solutions.

whenever possible.[25] Due to the high number of steps required to generate the results, software-based mathematical algorithms are used.[26] For instance, a nonrecombining binomial lattice with 1,000 steps has a total of 2×10^{301} nodal calculations to perform, making manual computation impossible without the use of specialized algorithms.[27] Figure 14.57 illustrates the calculation of convergence by using progressively higher lattice steps. The progression is based on sets of 120 steps (12 months per year multiplied by 10 years). The results are tabulated and the median of the average results is calculated. It shows that 4,200 steps is the best estimate in this customized binomial lattice, and this input is used throughout the analysis.[28]

Conclusion

It has been more than 30 years since Fisher Black, Myron Scholes, and Robert Merton derived their option pricing model and significant advancements have been made; therefore, do not restrict stock option pricing to one specific model (the BSM/GBM) while a plethora of other models and applications can be explored. The three mainstream approaches to valuing stock options are closed-form models (e.g., BSM, GBM, and American option approximation models), Monte Carlo simulation, and binomial lattices. The BSM and GBM will typically *overstate* the fair value of ESOs where there is suboptimal early exercise behavior coupled with vesting requirements and option forfeitures. In fact, firms using the BSM and GBM to value and expense ESOs may be *significantly* overstating their true expense. The BSM requires many underlying assumptions before it works and, as such, has significant limitations, including being applicable only for European options without dividends. In addition, American option approximation models are

Stock Price	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17
Strike Price	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17	$45.17
Maturity	10	10	10	10	10	10	10	10	10	10	10	10	10	10
Risk-Free Rate	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%	1.21%
Volatility	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.91%	49.941%	49.91%	49.91%	49.91%
Dividend	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Lattice Steps	10	50	100	120	600	1200	1800	2400	3000	3600	4200	4800	5400	6000
Suboptimal Behavior	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531	1.8531
Vesting	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Binomial Option Value	$20.55	$17.82	$17.32	$18.55	$17.55	$13.08	$13.11	$12.93	$12.88	$12.91	$13.00	$13.08	$12.93	$13.06

Segments	Steps	Results	Average
1	120	$18.55	$13.91
5	600	$17.55	$13.48
10	1200	$13.08	$13.00
15	1800	$13.11	$12.99
20	2400	$12.93	$12.97
25	3000	$12.83	$12.97
30	3600	$12.91	$12.99
35	4200	$13.00	$13.00
40	4800	$13.03	$13.02
45	5400	$12.93	$12.99
50	6000	$13.06	$13.06
		Median	$13.00

FIGURE 14.57 Convergence of the customized binomial lattice.

very complex and difficult to create in a spreadsheet. The BSM *cannot* account for American options, options based on stocks that pay dividends (the GBM model can, however, account for dividends in a European option), forfeitures, underperformance, stock price barriers, vesting periods, changing business environments and volatilities, suboptimal early exercise behavior, and a slew of other conditions. Monte Carlo simulation when used alone is another option valuation approach, but is restricted only to European options. Simulation can be used in two different ways: to solve the option's fair-market value through path simulations of stock prices, or used in conjunction with other approaches (e.g., binomial lattices and closed-form models) to capture multiple sources of uncertainty in the model.

Binomial lattices are flexible and easy to implement. They are capable of valuing American-type stock options with dividends but require computational power. Software applications should be used to facilitate this computation. Binomial lattices can be used to calculate American options paying dividends and can be easily adapted to solve ESOs with exotic inputs and used in conjunction with Monte Carlo simulation to account for the uncertain input assumptions (e.g., probabilities of forfeiture, suboptimal exercise behavior, vesting, underperformance) and to obtain a high precision at statistically valid confidence intervals. Based on the analyses throughout the case study, it is recommended that the use of a model that assumes an ESO is European style when, in fact, the option is American style with the other exotic variables should not be permitted, as this substantially overstates compensation expense. Many factors influence the fair-market value of ESOs, and a binomial lattice approach to valuation that considers these factors should be used. With due diligence, real-life ESOs can absolutely be valued using the customized binomial lattice approach as shown in this case study, where the methodology employed is pragmatic, accurate, and theoretically sound.