# Advanced Forecasting Techniques and Models: Spline Extrapolation

## Short Examples Series

## using

## Risk Simulator


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***Forecasting – Nonlinear Spline Extrapolation***

**File Name:** *Forecasting – Linear Interpolation*
**Location:** *Modeling Toolkit | Forecasting | Linear Interpolation* and *Modeling Toolkit | Yield Curve |Spline Interpolation and Extrapolation*
**Brief Description:** *This sample model illustrates how to compute a linear and nonlinear interpolation and extrapolation of missing values in a time-series forecast model and for forecasting*
**Requirements:** *Modeling Toolkit, Risk Simulator*
**Modeling Toolkit Function Used:** *B2InterpolationLinear, B2CubicSpline*

Sometimes interest rates or any type of time-dependent rates may have missing values. For instance, the Treasury rates for Years 1, 2, and 3 exist, and then jump to Year 5, skipping Year 4. We can, using linear interpolation (i.e., we assume the rates during the missing periods are linearly related), determine and "fill in" or interpolate their values. This model illustrates how to use this function (Figure 1).

In contrast, the cubic spline polynomial interpolation and extrapolation model is used to "fill in the gaps" of missing values (interpolation) and forecasting outside of the known values (extrapolation) when the underlying structure is nonlinear. For example, we can use apply this approach to spot yields and term structure of interest rates whereby the model can be used to both interpolate missing data points within a time series of interest rates (as well as other macroeconomic variables such as inflation rates and commodity prices or market returns) and also used to extrapolate outside of the given or known range, useful for forecasting purposes. Figure 2 illustrates the cubic spline model.

| Given Rates | |
|---|---|
| Time Period | Rates |
| 1 | 3.50% |
| 2 | 3.75% |
| 3 | 3.80% |
| 5 | 3.99% |
| 7 | 4.15% |
| 9 | 4.29% |
| 10 | 4.33% |
| 15 | 4.50% |
| 20 | 4.90% |
| 30 | 5.25% |

| Interpolated Rates | |
|---|---|
| Time Period | Rates |
| 1 | 3.50% |
| 2 | 3.75% |
| 3 | 3.80% |
| 4 | 3.90% |
| 5 | 3.99% |
| 6 | 4.07% |
| 7 | 4.15% |
| 8 | 4.22% |
| 9 | 4.29% |
| 10 | 4.33% |
| 11 | 4.36% |
| 12 | 4.40% |
| 13 | 4.43% |
| 14 | 4.47% |
| 15 | 4.50% |
| 16 | 4.58% |
| 17 | 4.66% |
| 18 | 4.74% |
| 19 | 4.82% |
| 20 | 4.90% |
| 21 | 4.94% |
| 22 | 4.97% |
| 23 | 5.01% |
| 24 | 5.04% |
| 25 | 5.08% |
| 26 | 5.11% |
| 27 | 5.15% |
| 28 | 5.18% |
| 29 | 5.22% |
| 30 | 5.25% |

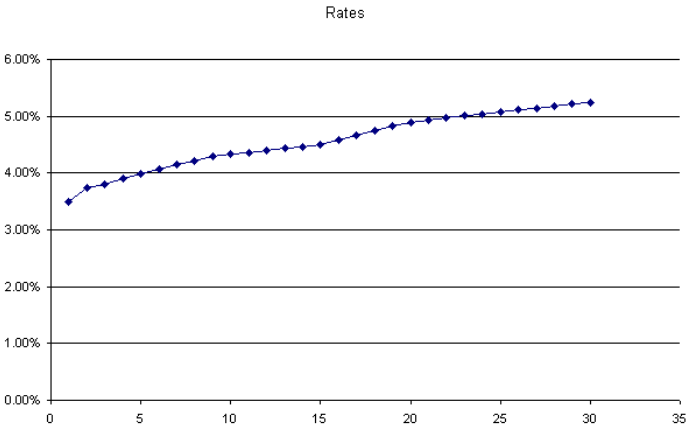


Figure 1: Interpolating missing values in a linear time-series

**YIELD CURVE - SPLINE INTERPOLATION MODEL**

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*Function: B2CubicSpline (Known X, Known Y, Required X)*

| Years | Spot Yields |
|---|---|
| 0.0833 | 4.55% |
| 0.2500 | 4.47% |
| 0.5000 | 4.52% |
| 1.0000 | 4.39% |
| 2.0000 | 4.13% |
| 3.0000 | 4.16% |
| 5.0000 | 4.26% |
| 7.0000 | 4.38% |
| 10.0000 | 4.56% |
| 20.0000 | 4.88% |
| 30.0000 | 4.84% |

These are the yields that are known and are used as inputs in the Cublic Spline Interpolation and Extrapolation model

**Spline Interpolation and Extrapolation Results**

| Years | Yield | Notes |
|---|---|---|
| 0.5 | 4.52% | Interpolate |
| 1.0 | 4.39% | Interpolate |
| 1.5 | 4.21% | Interpolate |
| 2.0 | 4.13% | Interpolate |
| 2.5 | 4.13% | Interpolate |
| 3.0 | 4.16% | Interpolate |
| 3.5 | 4.19% | Interpolate |
| 4.0 | 4.22% | Interpolate |
| 4.5 | 4.24% | Interpolate |
| 5.0 | 4.26% | Interpolate |
| 5.5 | 4.29% | Interpolate |
| 6.0 | 4.32% | Interpolate |
| 6.5 | 4.35% | Interpolate |
| 7.0 | 4.38% | Interpolate |
| 7.5 | 4.41% | Interpolate |
| 8.0 | 4.44% | Interpolate |
| 8.5 | 4.47% | Interpolate |
| 9.0 | 4.50% | Interpolate |

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Historical Data

XML This data is also available in XML format by clicking on the XML icon

Daily Treasury Yield Curve Rates
Daily Treasury Bill Rates
Daily Treasury Long-Term Rates
Daily Treasury Real Yield Curve Rates
Daily Treasury Real Long-Term Rates

**September 2007**

| Date | 1 mo | 3 mo | 6 mo | 1 yr | 2 yr | 3 yr | 5 yr | 7 yr | 10 yr | 20 yr | 30 yr |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 09/04/07 | 4.55 | 4.47 | 4.52 | 4.39 | 4.13 | 4.16 | 4.26 | 4.38 | 4.56 | 4.88 | 4.84 |
| 09/05/07 | 4.41 | 4.36 | 4.41 | 4.28 | 4.03 | 4.05 | 4.16 | 4.29 | 4.48 | 4.82 | 4.78 |
| 09/06/07 | 4.28 | 4.29 | 4.42 | 4.30 | 4.08 | 4.09 | 4.20 | 4.32 | 4.51 | 4.84 | 4.79 |
| 09/07/07 | 4.03 | 4.07 | 4.20 | 4.10 | 3.90 | 3.92 | 4.03 | 4.17 | 4.38 | 4.73 | 4.70 |
| 09/10/07 | 3.93 | 3.96 | 4.20 | 4.09 | 3.87 | 3.89 | 4.00 | 4.13 | 4.34 | 4.68 | 4.65 |
| 09/11/07 | 4.13 | 4.11 | 4.27 | 4.16 | 3.95 | 3.97 | 4.07 | 4.19 | 4.37 | 4.68 | 4.65 |
| 09/12/07 | 4.00 | 4.03 | 4.20 | 4.12 | 3.95 | 3.99 | 4.11 | 4.23 | 4.41 | 4.72 | 4.68 |
| 09/13/07 | 4.04 | 4.08 | 4.27 | 4.20 | 4.08 | 4.11 | 4.22 | 4.33 | 4.49 | 4.79 | 4.75 |
| 09/14/07 | 3.85 | 4.01 | 4.22 | 4.16 | 4.05 | 4.07 | 4.18 | 4.30 | 4.47 | 4.77 | 4.72 |
| 09/17/07 | 3.82 | 4.15 | 4.31 | 4.23 | 4.08 | 4.11 | 4.21 | 4.32 | 4.48 | 4.76 | 4.72 |
| 09/18/07 | 3.87 | 4.01 | 4.12 | 4.08 | 4.00 | 4.04 | 4.19 | 4.32 | 4.50 | 4.81 | 4.77 |

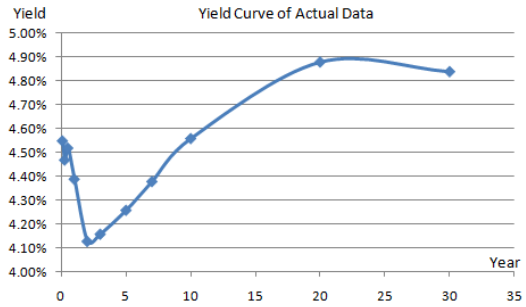



Figure 2 Interpolating missing values and forecast extrapolation in a nonlinear time-series

The cubic spline extrapolation approach can be applied either by using the Modeling Toolkit function or Risk Simulator's Cubic Spline module as seen in Figure 3. The example file can be accessed through **Risk Simulator | Examples | Advanced Forecasting Models** and going to the Cubic Spline worksheet. Figure 4 shows the results of the cubic spline results.

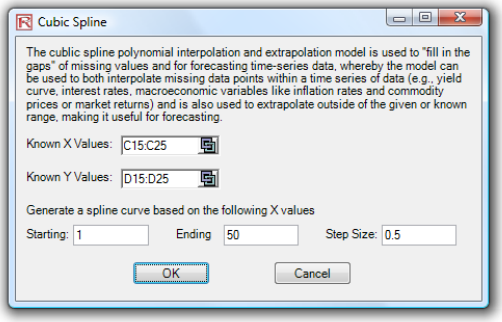
**Cubic Spline Interpolation and Extrapolation**

The cubic spline polynomial interpolation and extrapolation model is used to "fill in the gaps" of missing spot yields and term structure of interest rates whereby the model can be used to both interpolate missing data points within a time series of interest rates (as well as other macroeconomic variables such as inflation rates and commodity prices or market returns) and also used to extrapolate outside of the given or known range, useful for forecasting purposes.

| Years | Spot Yields |
|---|---|
| 0.0833 | 4.55% |
| 0.2500 | 4.47% |
| 0.5000 | 4.52% |
| 1.0000 | 4.39% |
| 2.0000 | 4.13% |
| 3.0000 | 4.16% |
| 5.0000 | 4.26% |
| 7.0000 | 4.38% |
| 10.0000 | 4.56% |
| 20.0000 | 4.88% |
| 30.0000 | 4.84% |

These are the yields that are known and are used as inputs in the Cubic Spline Interpolation and Extrapolation model

To run the Cubic Spline forecast, click on **Risk Simulator** | **Forecasting** | **Cubic Spline** and then click on the link icon and select C15:C25 as the Known X values (values on the x-axis of a time-series chart) and D15:D25 as the Known Y values (make sure the length of Known X and Y values are the same). Enter the desired forecast periods (e.g., Starting 1, Ending 50, Step Size 0.5). Click **OK** and review the generated forecasts and chart.

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Daily Treasury Yield Curve Rates
Daily Treasury Bill Rates
Daily Treasury Long-Term Rates
Daily Treasury Real Yield Curve Rates
Daily Treasury Real Long-Term Rates

**September 2007**

| Date | 1 mo | 3 mo | 6 mo | 1 yr | 2 yr | 3 yr | 5 yr | 7 yr | 10 yr | 20 yr | 30 yr |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 09/04/07 | 4.55 | 4.47 | 4.52 | 4.39 | 4.13 | 4.16 | 4.26 | 4.38 | 4.56 | 4.88 | 4.84 |
| 09/05/07 | 4.41 | 4.36 | 4.41 | 4.28 | 4.03 | 4.05 | 4.16 | 4.29 | 4.48 | 4.82 | 4.78 |
| 09/06/07 | 4.28 | 4.29 | 4.42 | 4.30 | 4.08 | 4.09 | 4.20 | 4.32 | 4.51 | 4.84 | 4.79 |
| 09/07/07 | 4.03 | 4.07 | 4.20 | 4.10 | 3.90 | 3.92 | 4.03 | 4.17 | 4.38 | 4.73 | 4.70 |
| 09/10/07 | 3.93 | 3.96 | 4.20 | 4.09 | 3.87 | 3.89 | 4.00 | 4.13 | 4.34 | 4.68 | 4.65 |
| 09/11/07 | 4.13 | 4.11 | 4.27 | 4.16 | 3.95 | 3.97 | 4.07 | 4.19 | 4.37 | 4.68 | 4.65 |
| 09/12/07 | 4.00 | 4.03 | 4.20 | 4.12 | 3.95 | 3.99 | 4.11 | 4.23 | 4.41 | 4.72 | 4.68 |
| 09/13/07 | 4.04 | 4.08 | 4.27 | 4.20 | 4.08 | 4.11 | 4.22 | 4.33 | 4.49 | 4.79 | 4.75 |
| 09/14/07 | 3.85 | 4.01 | 4.22 | 4.16 | 4.05 | 4.07 | 4.18 | 4.30 | 4.47 | 4.77 | 4.72 |
| 09/17/07 | 3.82 | 4.15 | 4.31 | 4.23 | 4.08 | 4.11 | 4.21 | 4.32 | 4.48 | 4.76 | 4.72 |
| 09/18/07 | 3.87 | 4.01 | 4.12 | 4.08 | 4.00 | 4.04 | 4.19 | 4.32 | 4.50 | 4.81 | 4.77 |

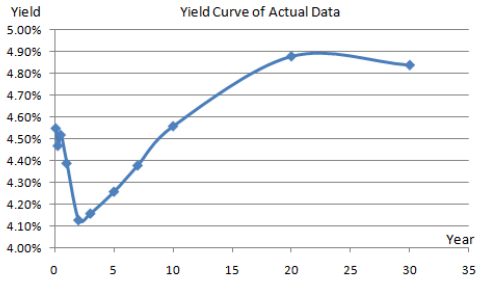


Figure 3: Cubic Spline forecast using Risk Simulator

## Cubic Spline Forecasts

The cubic spline polynomial interpolation and extrapolation model is used to "fill in the gaps" of missing values and for forecasting time-series data, whereby the model can be used to both interpolate missing data points within a time series of data (e.g., yield curve, interest rates, macroeconomic variables like inflation rates and commodity prices or market returns) and also used to extrapolate outside of the given or known range, making it useful for forecasting.

**Spline Interpolation and Extrapolation Results**

| X | Fitted Y | Notes |
|---|---|---|
| 1.0 | 4.39% | Interpolate |
| 2.0 | 4.13% | Interpolate |
| 3.0 | 4.16% | Interpolate |
| 4.0 | 4.22% | Interpolate |
| 5.0 | 4.26% | Interpolate |
| 6.0 | 4.32% | Interpolate |
| 7.0 | 4.38% | Interpolate |
| 8.0 | 4.44% | Interpolate |
| 9.0 | 4.50% | Interpolate |
| 10.0 | 4.56% | Interpolate |
| 11.0 | 4.61% | Interpolate |
| 12.0 | 4.66% | Interpolate |
| 13.0 | 4.70% | Interpolate |
| 14.0 | 4.74% | Interpolate |
| 15.0 | 4.77% | Interpolate |
| 16.0 | 4.80% | Interpolate |
| 17.0 | 4.83% | Interpolate |
| 18.0 | 4.85% | Interpolate |
| 19.0 | 4.87% | Interpolate |
| 20.0 | 4.88% | Interpolate |
| 21.0 | 4.89% | Interpolate |
| 22.0 | 4.89% | Interpolate |
| 23.0 | 4.89% | Interpolate |
| 24.0 | 4.89% | Interpolate |
| 25.0 | 4.89% | Interpolate |
| 26.0 | 4.88% | Interpolate |
| 27.0 | 4.87% | Interpolate |
| 28.0 | 4.86% | Interpolate |
| 29.0 | 4.85% | Interpolate |
| 30.0 | 4.84% | Interpolate |
| 31.0 | 4.83% | Extrapolate |
| 32.0 | 4.82% | Extrapolate |
| 33.0 | 4.81% | Extrapolate |
| 34.0 | 4.80% | Extrapolate |
| 35.0 | 4.79% | Extrapolate |
| 36.0 | 4.79% | Extrapolate |
| 37.0 | 4.79% | Extrapolate |
| 38.0 | 4.79% | Extrapolate |
| 39.0 | 4.79% | Extrapolate |
| 40.0 | 4.80% | Extrapolate |

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These are the known value inputs in the Cublic Spline Interpolation and Extrapolation model:

| Observation | Known X | Known Y |
|---|---|---|
| 1 | 0.0833 | 4.55% |
| 2 | 0.2500 | 4.47% |
| 3 | 0.5000 | 4.52% |
| 4 | 1.0000 | 4.39% |
| 5 | 2.0000 | 4.13% |
| 6 | 3.0000 | 4.16% |
| 7 | 5.0000 | 4.26% |
| 8 | 7.0000 | 4.38% |
| 9 | 10.0000 | 4.56% |
| 10 | 20.0000 | 4.88% |
| 11 | 30.0000 | 4.84% |

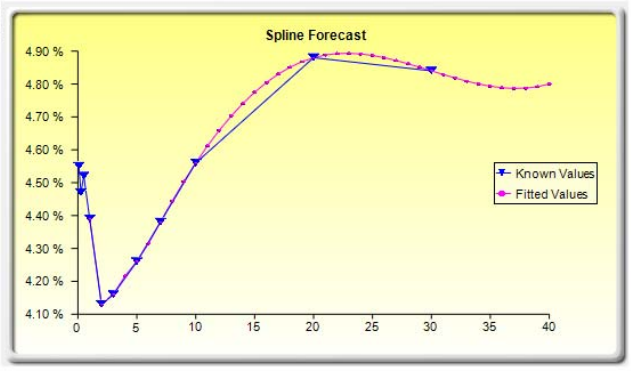


Figure 4: Cubic Spline forecast report