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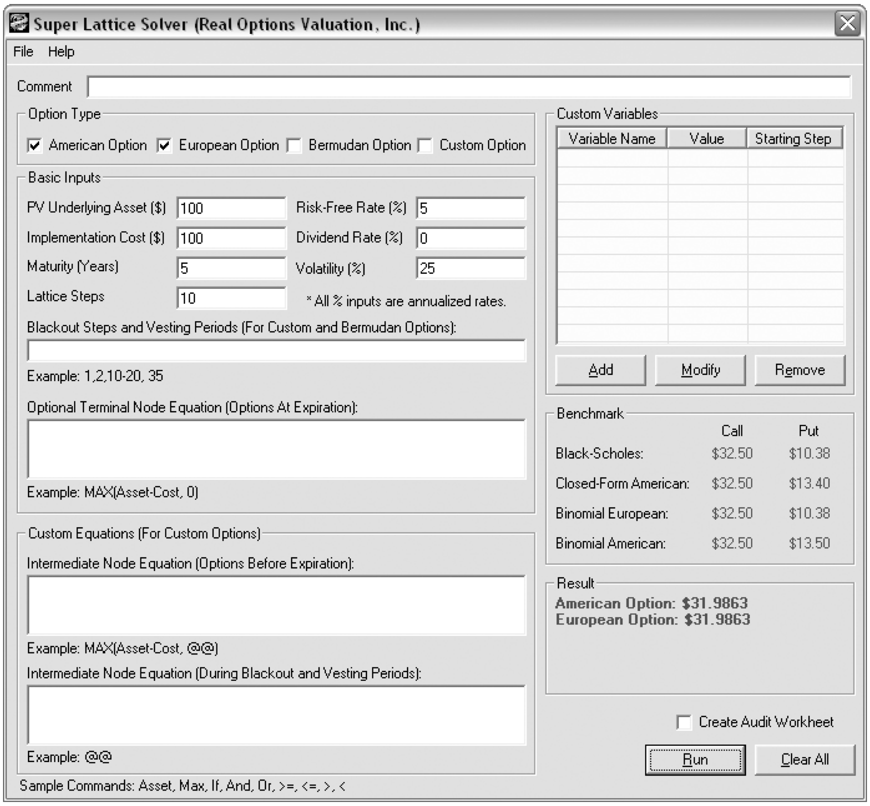

FIGURE 10.35 Ten-Step Binomial Lattice Comparison Result

AMERICAN AND EUROPEAN MEAN-REVERSION OPTION USING TRINOMIAL LATTICES

The *Mean-Reversion Option* in MNLS calculates both the American and European options when the underlying asset value is mean-reverting. A mean-reverting stochastic process reverts back to the long-term mean value (*Long-Term Rate Level*) at a particular speed of reversion (*Reversion Rate*). Examples of variables following a mean-reversion process include inflation rates, interest rates, gross domestic product growth rates, optimal production rates, price of natural gas, and so forth. Certain variables such as these succumb to either natural tendencies or economic/business conditions to revert to a long-term level when the actual values stray too far above or below this level. For instance, monetary and fiscal policy will prevent the economy from significant fluctuations, while policy goals tend to have a specific long-term target rate or level. Figure 10.36 illustrates a regular stochastic process (dotted line) versus a mean-reversion process (solid line). Clearly the mean-reverting process with its dampening effects will have a lower level of uncertainty than the regular process with the same volatility measure.

Figure 10.37 shows the call and put results from a regular option modeled using the Trinomial Lattice versus calls and puts assuming a mean-reverting (MR) tendency of the underlying asset using the Mean-Reverting Trinomial Lattice. Several items are worthy of attention:

- The MR call < regular call because of the dampening effect of the mean-reversion asset. The MR asset value will not increase as high as the regular asset value.
- Conversely, the MR put > regular put because the asset value will not rise as high, indicating that there will be a higher chance that the asset value will hover around the PV Asset, and a higher probability it will be below the PV Asset, making the put option more valuable.
- With the dampening effect, the MR call and MR put ($18.62 and $18.76) are more symmetrical in value than with a regular call and put ($31.99 and $13.14).
- The regular American call = regular European call because without dividends, it is never optimal to execute early. However, because of the mean-reverting tendencies, being able to execute early is valuable, especially before the asset value decreases. So, we see that MR American call > MR European call but of course both are less than the regular call.

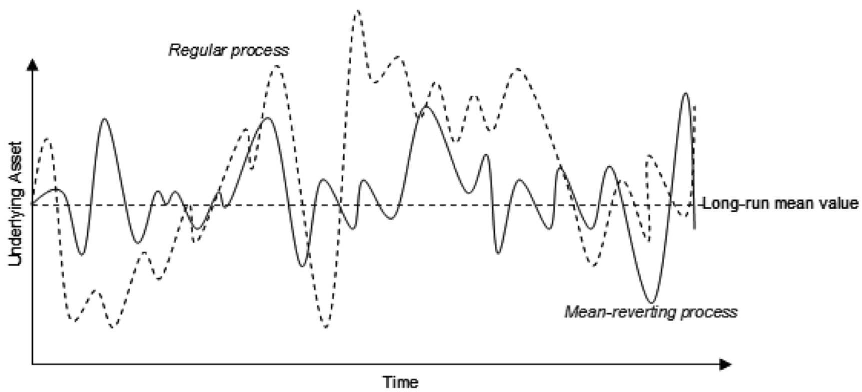

FIGURE 10.36 Mean-Reversion in Action

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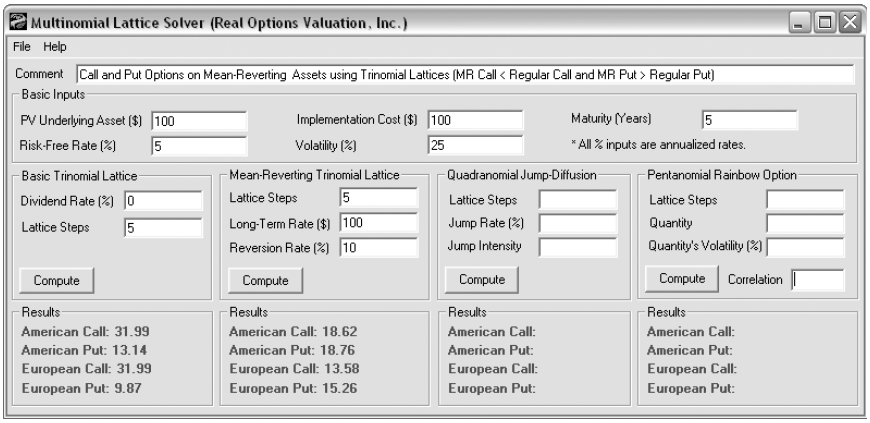

FIGURE 10.37 Comparing Mean-Reverting Calls and Puts to Regular Calls and Puts

Other items of interest in mean-reverting options include:

- The higher (lower) the long-term rate level, the higher (lower) the call options.
- The higher (lower) the long-term rate level, the lower (higher) the put options.

Finally, be careful when modeling mean-reverting options as higher lattice steps are usually required and certain combinations of reversion rates, long-term rate level, and lattice steps may yield unsolvable trinomial lattices. When this occurs, the MNLS will return error messages.

JUMP-DIFFUSION OPTION USING QUADRANOMIAL LATTICES

The *Jump-Diffusion Calls and Puts* for both American and European options applies the *Quadranomial Lattice* approach. This model is appropriate when the underlying variable in the option follows a jump-diffusion stochastic process. Figure 10.38 illustrates an underlying asset modeled using a jump-diffusion process. Jumps are commonplace in certain business variables such as price of oil and price of gas where prices take sudden and unexpected jumps (e.g., during a war). The underlying variable's frequency of jump is denoted as its *Jump Rate*, and the magnitude of each jump is its *Jump Intensity*.

The binomial lattice is only able to capture a stochastic process without jumps (e.g., Brownian Motion and Random Walk processes) but when there

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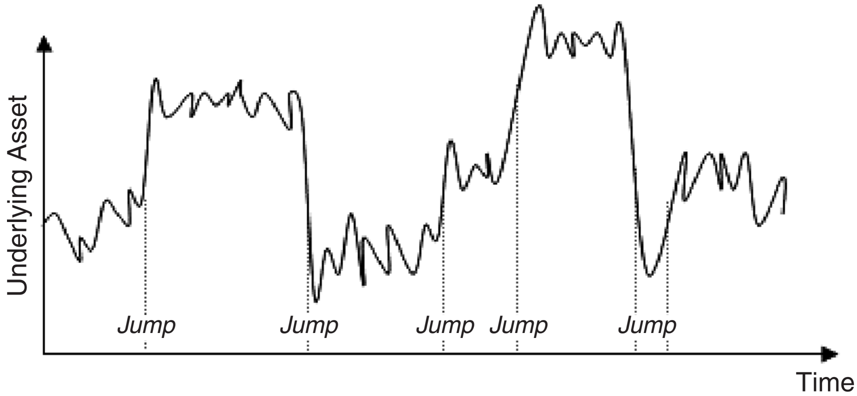

FIGURE 10.38 Jump-Diffusion Process

is a probability of jump (albeit a small probability that follows a Poisson distribution), additional branches are required. The quadranomial lattice (four branches on each node) is used to capture these jumps as seen in Figure 10.39.

Be aware that due to the complexity of the models, some calculations with higher lattice steps may take slightly longer to compute. Furthermore, certain combinations of inputs may yield negative implied risk-neutral probabilities and result in a noncomputable lattice. In that case, make sure the

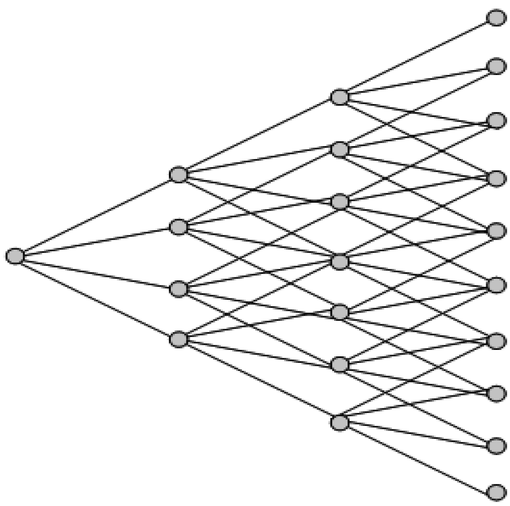

FIGURE 10.39 Quadranomial Lattice

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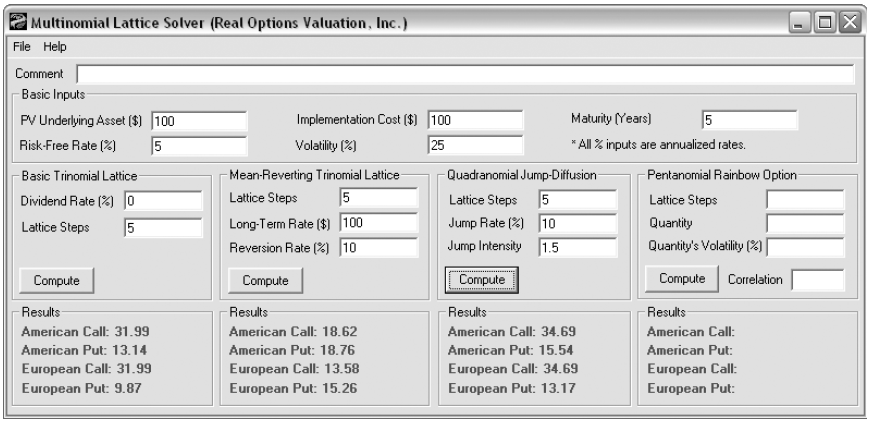

FIGURE 10.40 Quadranomial Lattice Results on Jump-Diffusion Options

inputs are correct (e.g., *Jump Intensity* has to exceed 1, where 1 implies no jumps; check for erroneous combinations of *Jump Rates*, *Jump Sizes*, and *Lattice Steps*). The probability of a jump can be computed as the product of the *Jump Rate* and time-step δt. Figure 10.40 illustrates a sample Quadranomial Jump-Diffusion Option analysis (example file used: *MNLS—Jump-Diffusion Calls and Puts Using Quadranomial Lattices*). Notice that the Jump-Diffusion call and put options are worth more than regular calls and puts, because with the positive jumps (10 percent probability per year with an average jump size of 1.50 times the previous values) of the underlying asset, the call and put options are worth more, even with the same volatility (i.e., $34.69 compared to $31.99 and $15.54 compared to $13.14).

DUAL-VARIABLE RAINBOW OPTION USING PENTANOMIAL LATTICES

The *Dual-Variable Rainbow Option* for both American and European options requires the *Pentanomial Lattice* approach. Rainbows on the horizon after a rainy day comprise various colors of the light spectrum, and although rainbow options aren't as colorful as their physical counterparts, they get their name from the fact that they have two or more underlying assets rather than one. In contrast to standard options, the value of a rainbow option is determined by the behavior of two or more underlying elements and by the correlation between these underlying elements. That is, the value of a rainbow

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option is determined by the performance of two or more underlying asset elements. This particular model is appropriate when there are two underlying variables in the option (e.g., *Price of Asset* and *Quantity*) where each fluctuates at different rates of volatilities but at the same time might be correlated (Figure 10.41). These two variables are usually correlated in the real world, and the underlying asset value is the product of price and quantity. Due to the different volatilities, a pentanomial or five-branch lattice is used to capture all possible combinations of products (Figure 10.42). Be aware that certain combinations of inputs may yield an unsolvable lattice with negative implied

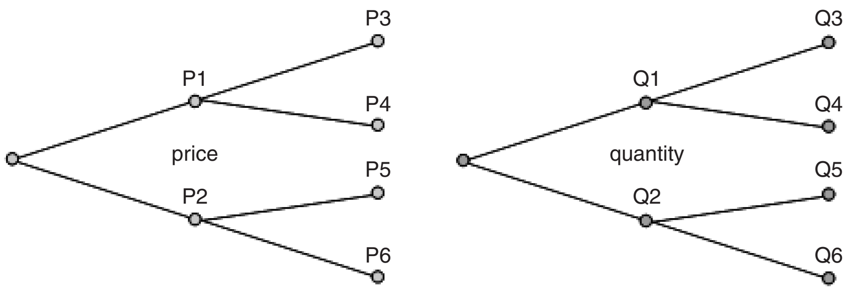

FIGURE 10.41 Two Binomial Lattices (Asset Prices and Quantity)

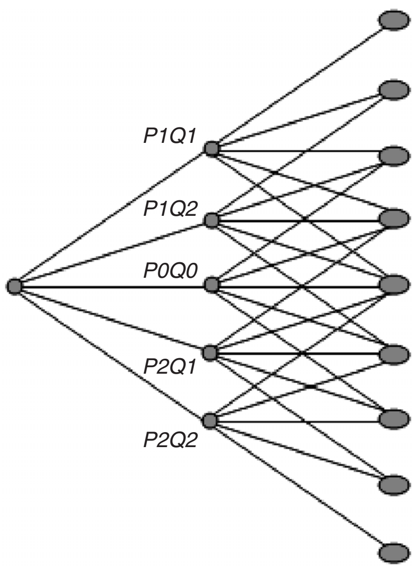

FIGURE 10.42 Pentanomial Lattice (Combining Two Binomial Lattices)

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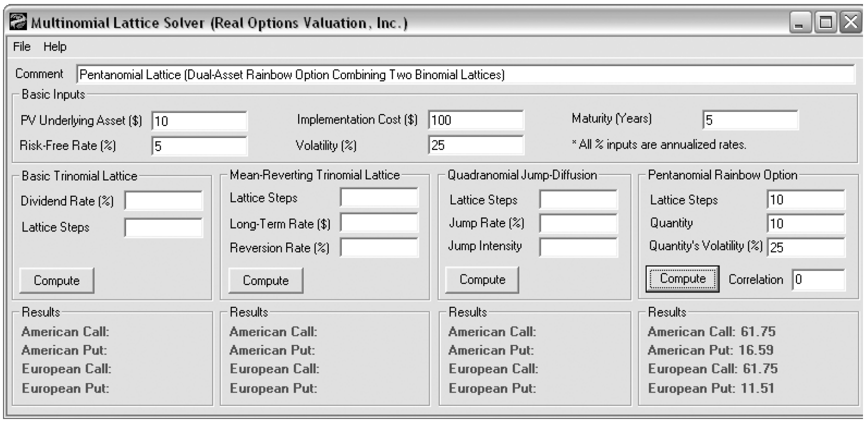

FIGURE 10.43 Pentanomial Lattice Solving a Dual-Asset Rainbow Option

probabilities. If that result occurs, a message will appear. Try a different combination of inputs as well as higher lattice steps to compensate.

Figure 10.43 shows an example Dual-Asset Rainbow Option (example file used: *MNLS—Dual-Asset Rainbow Option Pentanomial Lattice*). Notice that a high positive correlation will increase both the call option and put option values because if both underlying elements move in the same direction, there is a higher overall portfolio volatility (price and quantity can fluctuate at high-high and low-low levels, generating a higher overall underlying asset value). In contrast, negative correlations will reduce both the call option and put option values for the opposite reason due to the portfolio diversification effects of negatively correlated variables. Of course correlation here is bounded between –1 and +1 inclusive. If a real options problem has more than 2 underlying assets, either use the MSLS and/or Risk Simulator to simulate the underlying asset's trajectories and capture their interacting effects in a DCF model.

AMERICAN AND EUROPEAN LOWER BARRIER OPTIONS

The *Lower Barrier Option* measures the strategic value of an option (this applies to both calls and puts) that comes either in-the-money or out-of-the-money when the *Asset Value* hits an artificial *Lower Barrier* that is currently lower than the asset value. Therefore, a *Down-and-In* option (for both calls and puts) indicates that the option becomes live if the asset value hits the